The Importance of Commodity-Price Shocks

Samson M'boueke[†]

November 15, 2024

Click here for the most recent version

ONLINE APPENDIX

Abstract

Unexpected movements in world prices are believed to significantly affect domestic economic conditions in emerging market and developing economies (EMDEs). Previous studies have examined the role of world price fluctuations in EMDEs from an overall terms of trade perspective. In this paper, I depart from these studies by analyzing the importance of commodity import and export price shocks as a source of business cycles in EMDEs. I argue that for these countries, commodity price indices are better at capturing exogenous world price shocks than the overall terms of trade. Given the conflicting results between theoretical and empirical models in previous similar studies, I approach my analysis from both perspectives and test whether empirical and theoretical predictions can be reconciled. Specifically, I first use a Structural Vector Autoregressive (SVAR) model on 28 EMDEs to compute the shares of the variances of the trade balance, output, consumption, investment, and the real exchange rate attributable to commodity import and export price shocks. I find that empirically, these shocks explain 26-30% of business-cycle fluctuations in the selected countries. I then develop a 5-sector RBC model in which I explicitly introduce commodity import and export prices. Using this model, I compute the same variance shares as in the SVAR model. The RBC model predicts that on average, commodity import and export price shocks can only explain 2-3% of business-cycle fluctuations in the selected countries. In addition, a country-by-country comparison of the variance shares predicted by the SVAR and RBC models suggests that these models are also disconnected at the country level when it comes to measuring the importance of commodity-price shocks in EMDEs.

JEL No. C32, E32, F41, F44

Keywords: Commodity prices, Business cycles, Emerging market and developing economies.

[†]Samson M'boueke (smboueke@nd.edu). I would like to express my gratitude to my advisor, Joseph Kaboski, and my dissertation committee members, Nelson Mark and Robert Johnson, for their continuous support and feedback throughout the preparation of this paper. I am also grateful to the participants of the Development Lunch Seminar in the Economics Department at the University of Notre Dame for their valuable comments and suggestions.

1 Introduction

In recent years, there has been increasing recognition in international economics of the influence that world prices, particularly those of primary commodities, exert on the domestic economic conditions of emerging market and developing economies (EMDEs). Part of the reason is that these countries, often heavily reliant on a narrow export base centered around primary goods, are likely vulnerable to swings in the international prices of these goods. A number of studies, both empirical and theoretical, have examined this link between world price fluctuations and economic conditions in EMDEs from a terms of trade perspective. Specifically, these studies measure the importance of terms-of-trade shocks as a source of business-cycle fluctuations (Mendoza, 1995; Kose, 2002; Broda, 2004; Schmitt-Grohé and Martín Uribe, 2018), and find somewhat conflicting results.

The conventional view that emerges from calibrated real business cycle (RBC) models (Mendoza, 1995; Kose, 2002) is that a shock to the terms of trade explains a sizeable share (between 30% and 50%) of business-cycle fluctuations in EMDEs. However, recent empirical findings by Schmitt-Grohé and Martín Uribe (2018) on 38 EMDEs using a Structural Vector Autoregression (SVAR) model, paint a strikingly different picture. These findings indicate that terms-of-trade shocks account for less than 10% of movements in macroeconomic aggregates in EMDEs. Schmitt-Grohé and Martín Uribe (2018) also estimate key structural parameters of a three-sector RBC model similar to Mendoza (1995) and find that, although the importance assigned to terms-of-trade shocks by the RBC model is on average similar to that predicted by the empirical SVAR model, the predictions of the two models at the country level are far apart. They refer to this finding as the "terms-of-trade disconnect".

An important aspect of the analysis of terms-of-trade shocks in these papers centers on the measurement of the terms of trade. Traditionally, both theoretical and empirical studies have relied on the identification assumption that the terms of trade, often measured using the overall export-to-import price ratio, are exogenous to EMDEs. However, such terms of trade measures have been the subject of much criticism in the literature for their inability to accurately identify exogenous shocks to the terms of trade (Chen and Rogoff, 2003). In this paper, I argue that for EMDEs, country-specific commodity import and export prices are potentially a better proxy for how fluctuations in world prices affect these economies. The reasons are twofold: (1) Not only do EMDEs rely heavily on primary commodities for their export revenues, but many of these countries are also dependent on these commodities for import consumption. (2) Gruss and Kebhaj (2019) provide evidence that commodity price indices, largely determined in world markets, can be considered exogenous from the perspective of individual countries.

This paper, therefore, seeks to contribute to the analysis of the role of world price shocks in EMDEs from a commodity price perspective. Specifically, I ask how important commodity import and export price shocks are in explaining business-cycle fluctuations in EMDEs. Given the conflicting results in theoretical and empirical studies on similar questions, I approach my analysis from both perspectives and test whether empirical and theoretical predictions can be reconciled. More concretely, I first use a Structural Vector Autoregressive (SVAR) model on a set of 28 EMDEs to compute the shares of the variances of output, the trade balance, consumption, investment, and the real exchange rate attributable to commodity import and export price shocks. Building on Schmitt-Grohé and Martín Uribe (2018), I then develop a 5-sector RBC model that explicitly introduces commodity prices, which I use to predict the same variance shares as in the SVAR model. Finally, I calculate the correlations between the variance shares predicted by the SVAR and RBC models.

My paper is closely related to Schmitt-Grohé and Martín Uribe (2018) who analyze the importance of terms-of-trade shocks in the same fashion, and Di Pace, Juvenal, and Petrella (2020), who distinguish between the role of import and export prices to capture the relevance of terms of trade fluctuations for domestic business cycles in developing economies.

Findings from the SVAR model suggest that commodity export and import price shocks explain on average 26-30% of business-cycle fluctuations in the selected countries. This number is about 3 times larger than that of Schmitt-Grohé and Martín Uribe (2018) who focus on terms-of-trade shocks. However, according to the predictions of the 5-sector RBC model, these shocks explain only about 2-3% of business-cycle fluctuations on average. In addition, a country-by-country comparison of the variance shares predicted by both models indicates that there is a disconnect between the empirical and theoretical models when it comes to measuring the importance of commodity-price shocks in EMDEs. Overall, the correlation between the variance shares explained by the commodity import-price shocks is weakly negative. For the commodity export-price shocks, the correlation is almost zero.

The rest of the paper is structured as follows: Section 2 introduces the commodity price indices and documents a few empirical facts. Section 3 presents the SVAR analysis, Section 4 presents the theoretical analysis, and Section 5 provides concluding remarks.

2 The (Real) Commodity Price Indices

In the analysis of the effects of terms-of-trade shocks, the literature commonly assumes the exogeneity of the terms of trade for EMDEs. However, a significant challenge arises when using measures of the terms of trade based on overall export and import price indices. These indices, while widely used, pose difficulties in identifying exogenous terms-of-trade shocks because they do not only capture changes in world prices, but also other domestic factors (Chen and Rogoff, 2003; Gruss and Kebhaj, 2019).

To address this limitation, some strand of the literature suggests that commodity prices could offer a better proxy for the terms of trade in EMDEs compared to the overall indices of export and import prices commonly used in existing studies such as Mendoza (1995), Kose (2002), Broda (2004), and Schmitt-Grohé and Martín Uribe (2018). In line with this perspective, this paper focuses on analyzing the importance of commodity-price shocks as a source of business cycles in EMDEs. Although EMDEs are known for their reliance on primary commodities for export earnings, many of these countries also depend on commodity imports for final consumption. Therefore, I distinguish between the effects of commodity import and commodity export price shocks.

This section introduces the commodity export and import price indices used in my analysis. They come from a comprehensive database of country-specific commodity prices put together by Gruss and Kebhaj (2019), and are calculated as follows:

$$\Delta \log(\text{Index})_{i,t} = \sum_{j=1}^{J} \Omega_{i,j,t} \Delta P_{j,t} \tag{1}$$

where $P_{j,t}$ is the log of the real price of commodity j in year t, Δ denotes first differences, and $\Omega_{i,j,t}$ denote commodity and country-specific time-varying weights. For each commodity, real prices are constructed as the commodity price in US dollars deflated by the IMF's unit value index for manufactured exports. The log differences in Equation (1) are then used to generate the indices in levels, which are set to 2012 = 100. I denote the commodity export price index by $P_t^{x_c}$ and the commodity import price index by $P_t^{m_c}$.

The time-varying weights are based on average trade flows over the previous three calendar years, so that they reflect changes over time in the basket of commodities[1] traded but are predetermined vis-à-vis the price change in each year t:

$$\Omega_{i,j,t} = \frac{1}{3} \sum_{s=1}^{3} \omega_{i,j,t-s} \tag{2}$$

$\omega_{i,j,t} = x_{i,j,t} / \sum_{j=1}^{J} x_{i,j,t}$ for the export price index, and $\omega_{i,j,t} = m_{i,j,t} / \sum_{j=1}^{J} m_{i,j,t}$ for the import price index,

[1] The annual database of commodity price indices is based on 40 commodities broadly categorized in 4 groups: energy, metals, food and beverages, and agricultural raw materials. See Appendix A1.

where $x_{i,j,t}$ $(m_{i,j,t})$ denote the exports (imports) value of commodity j by country i in year t, expressed in US dollars.

To adjust for domestic inflation in my analysis, I express the commodity price indices in terms of domestic prices by dividing by the Consumer Price Index (P_t). So the price indices used throughout the paper are the "real" commodity export price index ($p_t^{x_c} = P_t^{x_c}/P_t$) and the "real" commodity import price index ($p_t^{m_c} = P_t^{m_c}/P_t$). This also ensures consistency with the theoretical model in Section 4, where prices are expressed in terms of domestic final goods. In the rest of the paper, I refer to the "real" commodity import and export price indices simply as commodity import and export price indices.

Empirical Regularities

I start by establishing three empirical facts about the commodity price indices as compared to two different terms of trade measures: (1) a commodity terms of trade (tot^c), calculated as the ratio of the commodity export price index to the commodity import price index, and the net barter terms of trade (tot^{nb}) used in Schmitt-Grohé and Martín Uribe (2018) and obtained from the World Development Indicators (WDI). The latter is a measure of the overall export-to-import price ratio.

FACT 1: *In EMDEs, commodity export and import price shocks are larger than commodity terms-of-trade shocks, which in turn are larger than net barter terms-of-trade shocks.*

Figure (1) provides pairwise comparisons of the volatility of commodity prices and terms of trade, measured by the standard deviation of their cyclical components. The last two scatterplots comparing commodity prices and commodity terms of trade show that the overwhelming majority of points fall below the 45° line. In addition, the first two scatterplots comparing the net barter terms of trade and commodity prices show that the entire cloud of points, except one or two points, lies above the 45° line. These facts indicate that commodity price indices, taken individually, are more volatile than terms of trade measures calculated using export/import price ratios. Therefore, we might expect commodity price indices to play a bigger role in driving business cycles than terms of trade measures. The middle-left scatterplot shows that commodity terms of trade are more volatile than net barter terms of trade, and the middle-right scatterplot shows that commodity import prices are almost as volatile as commodity export prices, although the former are slightly more volatile.

FACT 2: *In EMDEs, commodity export and import price shocks are highly persistent. Commodity terms-of-trade shocks are also highly persistent, but less so than commodity export and import price shocks. Net barter terms-of-trade shocks are only moderately persistent.*

Similarly to Figure (1), Figure (2) presents pairwise comparisons of the persistence of commodity prices and terms of trade, which is measured by the first-order autocorrelation of their cyclical components. A similar interpretation of the scatterplots as previously suggests that the first-order autocorrelations of the commodity price indices are overwhelmingly larger than those of the commodity terms of trade, which in turn are larger than those of the net barter terms of trade. Finally, commodity import price shocks are almost as persistent as commodity export price shocks, although the former are slightly more persistent.

FACT 3: *In EMDEs, commodity export and import price shocks are weakly procyclical, whereas commodity and net barter terms of trade shocks are acyclical.*

Table (1) presents the median correlations of the cyclical components of commodity price indices and terms of trade with the cyclical component of GDP. For the median country, commodity import and export prices are weakly positively correlated with GDP (corr = 0.39), and there is almost no correlation between commodity/net barter terms of trade and GDP. We do observe substantial cross-country dispersion in the correlations as indicated by the median absolute deviations.

Together, these facts suggest that commodity prices display different cyclical dynamics than terms of trade, especially the net barter terms of trade. This seems to make sense because of the reliance of EMDEs on primary commodities, both for their export revenues and import consumption. In addition, the fact that commodity import and export prices have closely aligned cyclical dynamics is likely due to the focus on commodities. EMDEs also depend heavily on the import of manufactured goods, so the inclusion of these goods in the calculation of the import price index will likely result in different export and import price dynamics.

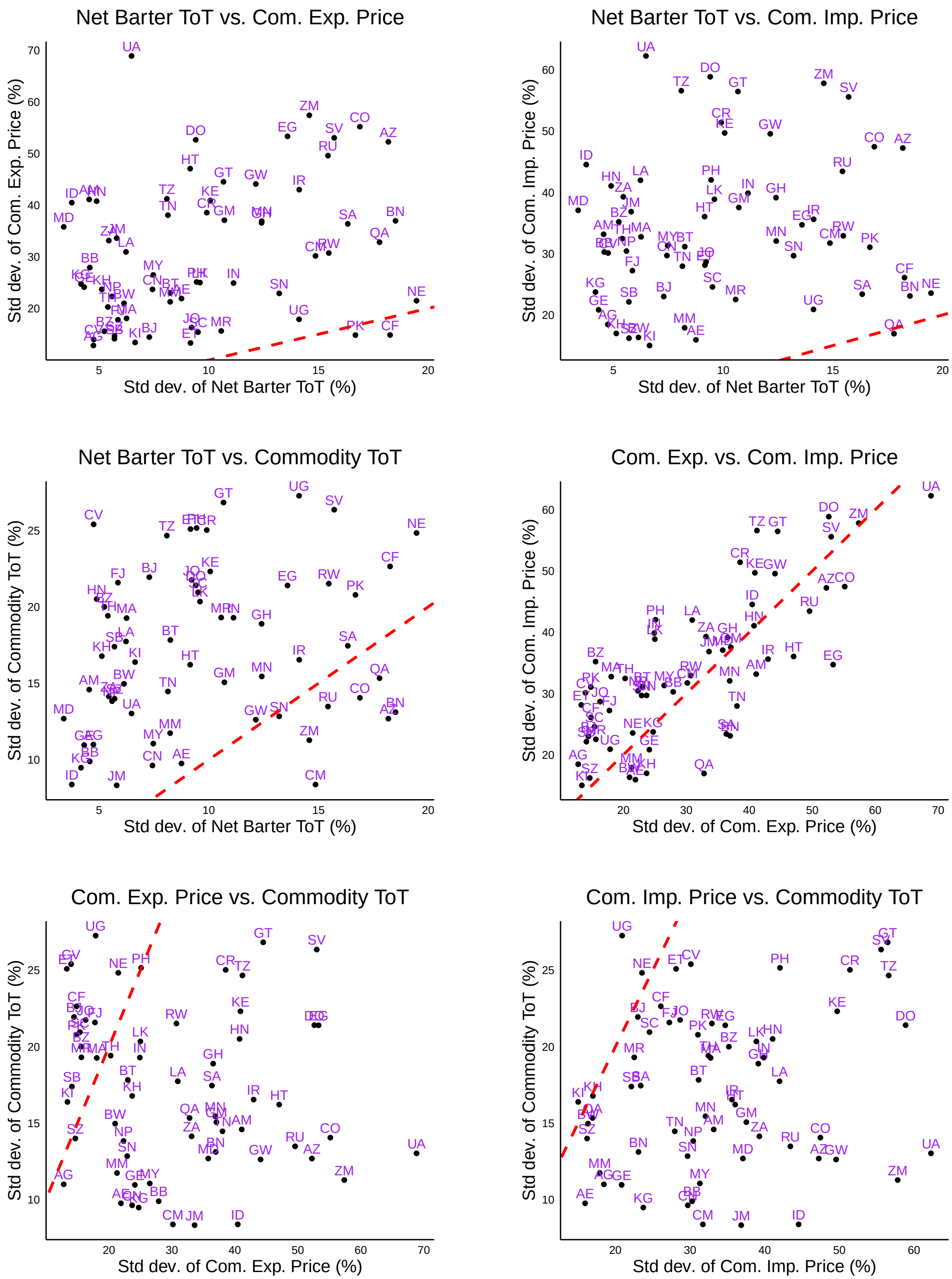

Figure 1: Comparison of the volatility of commodity prices and terms of trade. All variables are log-quadratically detrended, so the standard deviations are those of their cyclical components, that is, the standard deviations of the percent deviations from trend. Countries retained are those with the longest uninterrupted data spans between 1980 and 2019. In addition, extreme values are filtered out by retaining the values within the 10th and 90th percentiles. The final number of countries is 64. The dashed red line is the 45° line.

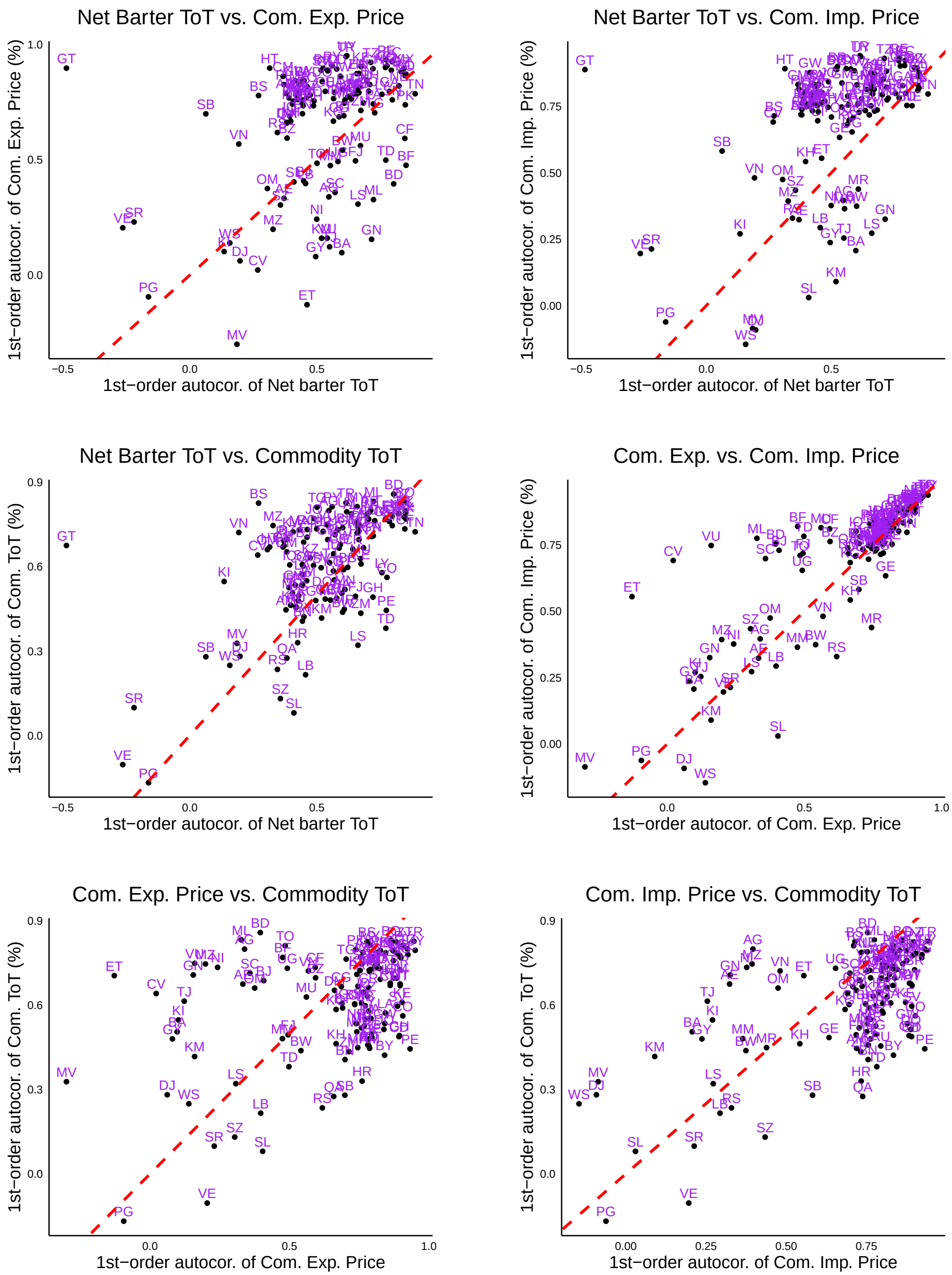

Figure 2: Comparison of the persistence of commodity prices and terms of trade. All variables are log-quadratically detrended, so the first-order autocorrelations are those of their cyclical components, that is, the first-order autocorrelations of the percent deviations from trend. Countries retained are those with the longest uninterrupted data spans between 1980 and 2019. There are 125 such countries. The dashed red line is the 45° line.

Table 1: Cyclicality of commodity prices and terms of trade

	$\text{cor}(\widehat{p}_t^{m_c}, \widehat{y}_t)$	$\text{cor}(\widehat{p}_t^{x_c}, \widehat{y}_t)$	$\text{cor}(\widehat{tot}_t^{c}, \widehat{y}_t)$	$\text{cor}(\widehat{tot}_t^{nb}, \widehat{y}_t)$
Median	0.39	0.39	0.05	0.14
Median absolute deviation	0.54	0.49	0.53	0.35

Notes: "cor" stands for correlation. $\widehat{y}_t$, $\widehat{p}_t^{m_c}$, $\widehat{p}_t^{x_c}$, $\widehat{tot}_t^{c}$, and $\widehat{tot}_t^{nb}$ denote the cyclical components of GDP, commodity import price index, commodity export price index, commodity terms of trade, and net barter terms of trade, respectively. All variables are log-quadratically detrended, so the correlations are those between the cyclical components of the variables. Countries retained are those with the longest uninterrupted data spans between 1980 and 2019. There are 125 such countries.

3 The Importance of Commodity-Price Shocks: SVAR Analysis

The objective of this paper is to quantify the importance of commodity import and export price shocks in generating business cycle fluctuations in EMDEs. In papers where a similar analysis is carried out with a focus on terms-of-trade shocks, it seems to matter whether the question is approached from an empirical or a theoretical perspective. In this paper, I closely follow Schmitt-Grohé and Martín Uribe (2018) in trying to reconcile empirical and theoretical predictions on the role of commodity import and export price shocks in EMDEs.

I start with empirical predictions using a Structural Vector Autoregressive (SVAR) model. As in much of the literature, the main measure of the importance of commodity price shocks is the shares of the variances of the dependent variables of interest attributable to these shocks. The sample of countries used for the rest of the paper is more restricted than in the previous data analysis. For comparability purposes, the starting point is the sample of 38 poor and emerging economies used in Schmitt-Grohé and Martín Uribe (2018). Their data span the period from 1980 to 2011, using the September 2012 vintage of the World Development Indicators (WDI). I extend the sample to 2019 using the September 2023 vintage of WDI. In the latter version, Korea, which was considered an emerging economy in Schmitt-Grohé and Martín Uribe (2018), is a developed economy and therefore excluded from the sample. In addition, some countries have too many missing data for at least one of the variables of interest and are therefore excluded from the sample as well. The final list of countries retained in the September 2023 vintage is the one where countries have at least 30 consecutive observations from 1980 to 2019. There are 28 such countries, namely Algeria, Brazil, Burundi, Cameroon, Central African Republic, Colombia, Costa Rica, Dominican Republic, Egypt, El Salvador, Ghana, Guatemala, India, Jordan, Kenya, Madagascar, Malaysia, Mauritius, Mexico, Morocco, Pakistan, Paraguay, Peru, Philippines, Senegal, South Africa, Thailand, and Türkiye.

3.1 The SVAR Model

The setup of the model is the same as the SVAR model with interest-rate spreads in Schmitt-Grohé and Martín Uribe (2018). The model includes seven variables, namely the commodity import or export price index ($p_t^{x_c}$ or $p_t^{m_c}$), the trade balance (tb_t), real output per capita (y_t), real private consumption per capita (c_t), real gross investment per capita (i_t), the dollar real exchange rate ($rer_t = \frac{\mathcal{E}_t P_t^{US}}{P_t}$), and the U.S. interest-rate spread (s_t). $\mathcal{E}_t$ denotes the dollar nominal exchange rate, given by the domestic-currency price of one U.S. dollar; P_t^{US} denotes the U.S. consumer price index, and P_t denotes the domestic consumer price index.

The U.S. interest-rate spread, defined as Moody's seasoned Baa corporate bond yield minus the Federal Funds rate, is included in the model because a number of studies have shown that world interest-rate shocks are a source of business cycles in emerging and developing economies (e.g., Garcia-Cicco, Pancrazi, and Martin Uribe (2010), Fernández-Villaverde et al. (2011), and Akıncı (2013)). All variables, except the trade balance, are log-quadratically detrended. The trade balance is first divided by the trend component of output, and the resulting ratio is quadratically detrended. The empirical model is of the form:

$$\mathbf{A}_0 \mathbf{x}_t = \mathbf{A}_1 \mathbf{x}_{t-1} + \mathbf{u}_t, \tag{3}$$

where the vector $\mathbf{x}_t$ is given by $\mathbf{x}_t \equiv (\widehat{p_t}^k, \widehat{s}_t, \widehat{tb}_t, \widehat{y}_t, \widehat{c}_t, \widehat{i}_t, \widehat{rer}_t)'$, with $k = x_c$ or m_c. Hatted variables, except $\widehat{tb}_t$, denote log deviations of the variables from their time trends. $\widehat{tb}_t$ denotes the deviation from trend of the ratio of the trade balance to trend output. $\mathbf{A}_0$ and $\mathbf{A}_1$ are 7-by-7 matrices of coefficients, and $\mathbf{A}_0$ is assumed to be lower triangular with 1 on the main diagonal. $\mathbf{u}_t$ is a 7-by-1 random vector with mean 0 and diagonal variance-covariance matrix $\mathbf{\Sigma}$. Pre-multiplying the system by $\mathbf{A}_0^{-1}$, we can write:

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{\Pi}\boldsymbol{\epsilon}_t \tag{4}$$

where

$$\mathbf{A} \equiv \mathbf{A}_0^{-1}\mathbf{A}_1, \quad \mathbf{\Pi} \equiv \mathbf{A}_0^{-1}\mathbf{\Sigma}^{1/2}, \quad \text{and} \quad \boldsymbol{\epsilon}_t \equiv \mathbf{\Sigma}^{-1/2}\mathbf{u}_t$$

The vector $\boldsymbol{\epsilon}_t$ is a random variable with mean 0 and identity variance-covariance matrix. The key identifying assumption is that the typical emerging or developing economy takes commodity import and export prices and the U.S. interest-rate spread as exogenously given. Formally, the commodity import or export price index and the U.S. interest-rate spread follow the joint law of motion:

$$\begin{bmatrix} \widehat{p}_t^k \\ \widehat{s}_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \widehat{p}_{t-1}^k \\ \widehat{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \pi_{11} & 0 \\ \pi_{21} & \pi_{22} \end{bmatrix} \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix} \tag{5}$$

where ϵ_t^1 is the commodity import or export price shock, and ϵ_t^2 is the interest-rate spread shock. The ordering in Eq (5) gives the commodity price indices the highest chance of being an important source of fluctuations in domestic variables, because any innovation in the commodity price indices is due to commodity price shocks.[2] The ordering of elements 3 to 7 of $\mathbf{x}_t$ in the SVAR is immaterial because the analysis focuses on the effects of commodity-price (and interest rate-spread) shocks.

Finally, the model is estimated country-by-country by OLS. In the next sub-section, I report the median impulse responses to a 10% innovation in the commodity import and export price indices, as well as the shares of the variances explained by these innovations.

3.2 Impulse Responses

Figure (3) displays the median impulse responses to commodity import and export price shocks. The online Appendix provides country-specific impulse responses with 66% confidence bands. On impact, a 10% improvement in commodity import/export prices decreases the trade balance by 0.4% of GDP and 0.2% of GDP, respectively. This is in stark contrast with the Harberger–Laursen–Metzler (HLM) effect for terms-of-trade shocks, where an improvement in the terms of trade increases the trade balance. At the individual country level, this negative effect holds for 23 over 28 countries for the commodity import-price shock, and 17 over 28 countries for the commodity export-price shock. Interestingly, both shocks have expansionary effects on aggregate activity, despite the reduction in the trade balance. Specifically, output increases by 0.24% and 0.3% respectively on impact, and reaches its maximum increase at 0.32% above trend two years after the commodity import price shock, and at 0.43% one year after the commodity export price shock. On impact, investment and private consumption increase by 1.55% and 0.56% respectively for the commodity import-price shock, and by 1.4% and 0.22% respectively for the commodity export-price shock. The real exchange rate appreciates on impact by 0.8% and 0.25%, respectively.

The country-by-country impulse responses in the online Appendix show, however, that there is some dispersion both within and across countries in terms of the direction, magnitude, and statistical significance of the impulse responses.

[2] The results are robust to an alternative specification where the spread shock is placed first in the SVAR model

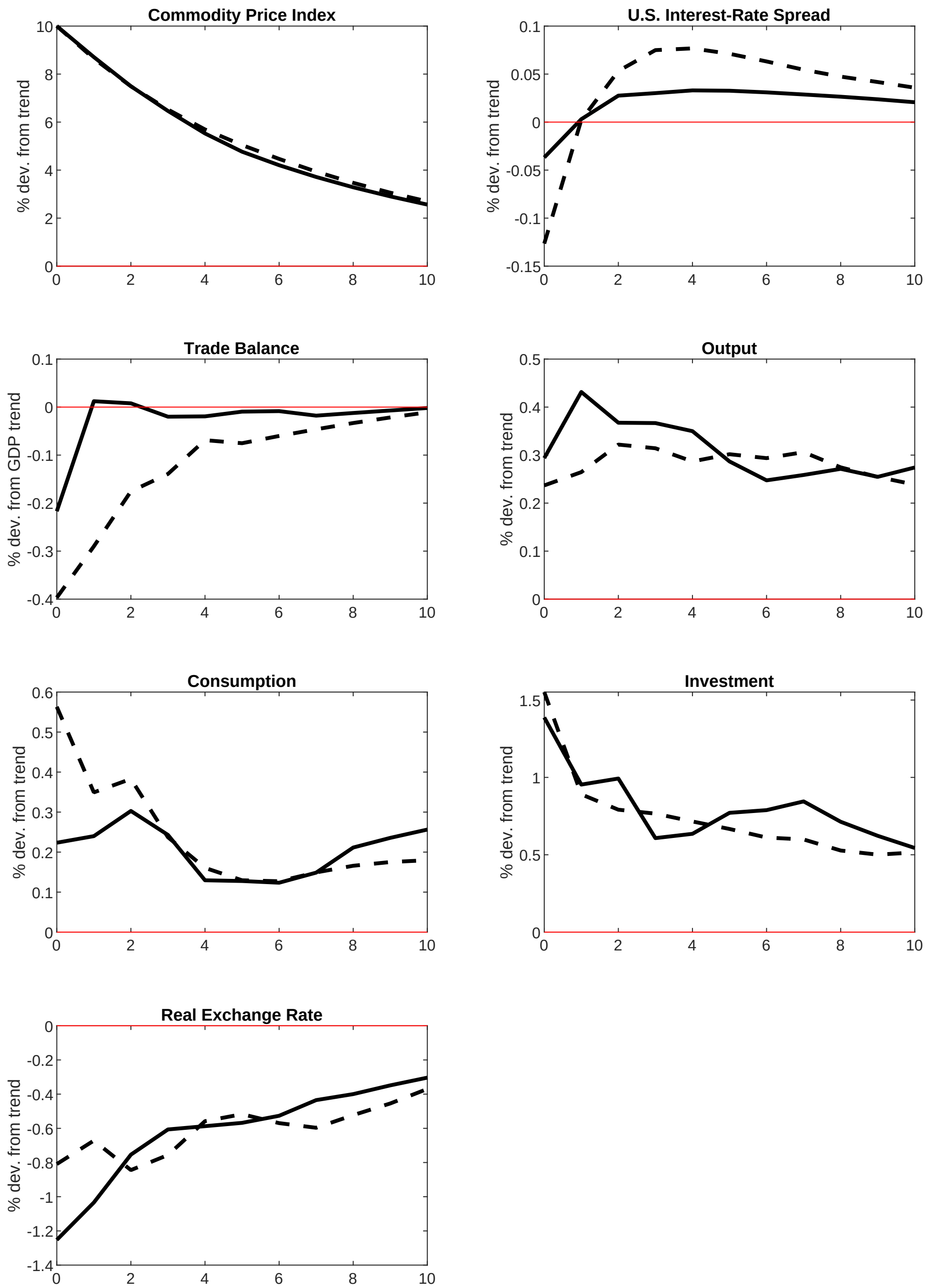

Figure 3: Impulse responses to commodity-price shocks. The solid line represents responses to commodity export-price shocks, and the dashed line represents responses to commodity import-price shocks. The x-axis measures years after the shock. The data covers 28 EMDEs from 1980 to 2019. Impulse responses are point-by-point medians across countries. The online appendix presents country-specific impulse responses with 66% confidence bands.

3.3 Variance Decomposition

Table (2) displays the shares of the variances of the seven variables in the SVAR explained by commodity import and export price shocks. Focusing on the business-cycle variables of interest, that is, the trade balance, output, consumption, investment, and the real exchange rate, the median variance shares across countries range between 20.65% and 33.55% for the commodity export price shocks, and the average of these medians is about 26%. For the commodity import price shocks, the median variance shares range between 25.42% and 33.37%, with an average of about 30%. These average numbers are about 3 times larger than the findings in Schmitt-Grohé and Martín Uribe (2018) for terms-of-trade shocks. My results are also consistent with the conventional view that world price swings play a major role in explaining business-cycle fluctuations in EMDEs.

Table 2: Shares of variances explained by commodity-price shocks: SVAR Evidence

	Commodity export-price shocks							Commodity import-price shocks						
Country	$\widehat{p}^{x_c}$	$\widehat{s}$	$\widehat{tb}$	$\widehat{y}$	$\widehat{c}$	$\widehat{i}$	$\widehat{rer}$	$\widehat{p}^{m_c}$	$\widehat{s}$	$\widehat{tb}$	$\widehat{y}$	$\widehat{c}$	$\widehat{i}$	$\widehat{rer}$
Algeria	77.17	6.35	12.94	8.43	14.77	2.09	68.83	99.71	6.63	27.81	28.14	8.02	39.40	64.57
Brazil	95.16	7.68	4.42	58.27	30.77	55.43	15.97	93.96	8.57	3.66	60.87	33.22	56.35	14.68
Burundi	87.13	4.40	5.44	10.17	6.53	4.92	2.35	99.49	9.07	2.86	12.12	13.80	8.97	1.16
Cameroon	99.82	5.59	13.39	26.42	28.88	25.08	20.64	99.66	8.46	6.19	34.69	34.17	32.16	18.15
Central Afr. Rep.	85.11	2.29	17.19	44.30	42.86	21.31	24.74	94.34	3.92	5.67	46.26	46.17	22.72	22.43
Colombia	64.28	2.37	14.67	18.57	29.94	23.90	6.97	88.70	8.83	41.01	36.03	27.19	38.65	3.58
Costa Rica	89.17	0.46	20.68	63.99	44.31	22.93	23.37	98.70	5.41	34.76	54.03	65.84	15.26	21.14
Dominican Rep.	99.97	11.72	62.47	63.96	48.71	68.17	34.29	95.92	7.74	23.92	5.97	20.98	29.54	9.16
Egypt	99.77	9.08	21.88	15.48	15.81	17.66	16.19	98.77	8.03	25.86	20.24	8.61	20.55	21.82
El Salvador	93.77	12.13	31.73	27.82	9.77	38.15	70.30	99.38	7.39	18.65	21.58	27.60	33.06	75.35
Ghana	74.78	2.21	24.38	41.29	26.75	7.19	43.00	99.44	5.07	16.11	9.32	23.75	12.13	34.15
Guatemala	94.01	5.22	59.22	4.68	16.03	32.27	57.87	99.73	8.31	44.10	6.79	16.81	38.24	49.90
India	82.59	1.84	6.06	5.98	2.94	17.27	3.80	98.42	6.46	71.22	83.42	74.25	53.16	46.91
Jordan	89.14	0.74	20.62	60.90	13.03	42.41	20.68	99.55	8.96	29.10	54.23	15.38	45.14	32.17
Kenya	91.79	2.02	49.39	34.07	26.69	67.24	24.88	98.57	7.71	74.29	21.00	57.22	69.63	57.49
Madagascar	84.01	1.44	7.21	33.03	6.21	9.06	26.71	99.92	7.01	11.37	29.13	8.84	33.68	42.70
Malaysia	84.88	9.44	21.74	1.15	8.90	14.31	26.69	94.02	4.62	29.17	0.42	40.20	6.25	18.97
Mauritius	99.28	5.35	25.41	38.37	12.82	11.99	30.73	99.73	4.81	30.05	39.96	26.41	21.92	45.49
Mexico	91.63	2.97	15.70	14.42	12.13	15.62	50.17	93.16	4.91	37.50	29.85	34.66	35.01	60.07
Morocco	96.75	2.47	23.71	73.43	46.45	44.13	51.95	98.04	2.64	13.57	72.38	43.91	39.82	53.37
Pakistan	90.48	5.99	11.74	64.48	36.61	25.41	14.18	99.99	12.63	24.97	24.39	1.03	8.65	4.11
Paraguay	92.42	9.78	29.25	22.74	24.76	15.60	43.23	97.82	6.81	44.69	31.07	38.52	22.11	34.94
Peru	99.19	3.92	16.97	65.96	66.66	60.79	64.64	99.67	6.71	21.31	40.72	46.78	43.27	49.59
Philippines	99.98	6.91	26.00	1.05	2.09	2.08	1.51	99.90	10.00	31.44	1.38	4.05	7.40	1.21
Senegal	84.82	2.47	34.53	65.01	58.47	59.33	54.66	99.96	6.12	20.07	24.79	22.47	20.40	6.34
South Africa	73.70	0.47	43.36	28.86	19.76	44.22	56.68	97.90	5.20	45.80	19.93	32.08	36.77	50.75
Thailand	85.42	2.47	8.52	52.54	30.92	42.26	35.85	94.93	2.76	10.82	48.74	31.84	34.59	44.85
Türkiye	99.41	6.06	85.76	7.27	13.41	64.12	69.01	91.08	5.39	18.49	85.51	83.35	75.83	4.06
Median	90.48	4.40	20.65	33.55	25.72	24.49	26.70	98.73	6.76	25.42	29.49	29.72	33.37	33.16
Median abs. dev.	5.17	2.16	7.49	24.05	13.25	16.36	16.42	0.99	1.62	10.60	14.00	13.55	11.36	17.16

Notes: Shares are expressed in percent. The data covers 28 EMDEs from 1980 to 2019.

To find out how the importance of commodity price shocks compares to that of terms-of-trade shocks in the SVAR model, I also produce the median variance shares of the variables of interest attributable to a commodity terms-of-trade shock and a net barter terms-of-trade shock. The results are presented in Table (3). On average, the commodity terms-of-trade shock explains 20% of business-cycle fluctuations, while the net barter terms-of-trade shock explains only 14%. These results are consistent with the empirical facts established in Section 2.

Table 3: Median shares of variances explained by commodity-price shocks and terms-of-trade shocks.

	$\widehat{tb}$	$\widehat{y}$	$\widehat{c}$	$\widehat{i}$	$\widehat{rer}$
Commodity import-price shocks	25.42	29.49	29.72	33.37	33.16
Commodity export-price shocks	20.65	33.55	25.72	24.49	26.70
Commodity terms-of-trade shocks	23.51	14.50	21.07	20.75	17.77
Net barter terms-of-trade shocks	20.07	12.34	14.13	11.02	14.18

Notes: Shares are expressed in percent. The data covers 28 EMDEs from 1980 to 2019.

The SVAR model implies that commodity import and export price shocks explain on average 26-30% of business-cycle fluctuations in our sample of 28 EMDEs, which are pretty large numbers. Calibrated RBC models on the role of terms-of-trade shocks, e.g. Mendoza (1995), typically arrive at the conclusion that these shocks explain more than 30% of business cycles in EMDEs. Can these models predict the same patterns for commodity price shocks? In order words, can they match the findings from the empirical SVAR model? The next section addresses this question.

4 The Theoretical Model

To gauge the importance of commodity-price shocks from a theoretical standpoint, I build on Schmitt-Grohé and Martín Uribe (2018)'s 3-sector real business-cycle model, which itself is similar in structure to Mendoza (1995). Schmitt-Grohé and Martín Uribe (2018)'s model, referred to as the "MXN" model, includes an importable sector (the m sector), an exportable sector (the x sector), and a non-tradable sector (the n sector). Because my analysis focuses on commodity prices, I further divide the importable and exportable sectors into commodity (c) and non-commodity ($\bar{c}$) sub-sectors. This results in a modified MXN model with 5 sectors, namely the commodity importable sector (m_c), the non-commodity importable sector ($m_{\bar{c}}$), the commodity exportable sector (x_c), the non-commodity exportable sector ($x_{\bar{c}}$), and the non-tradable sector (n). I refer to this modified model as the MXN-C model, where "C" stands for commodity. The goal is to explicitly introduce commodity prices into the MXN model. All other features of the MXN model are preserved: (1) Employment in all sectors can vary endogenously over the business cycle; (2) capital accumulation is allowed in all sectors; and (3) investment goods are not fully imported and can have nontraded components.

4.1 Households

The model economy consists of a large number of identical households whose preferences are described by the period utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, (h_t^j)_{j\in S}\right), \qquad S = \{m_c, m_{\bar{c}}, x_c, x_{\bar{c}}, n\} \tag{6}$$

where c_t denotes consumption, and h_t^j stands for hours worked in sector j. The utility function is assumed to display constant relative risk aversion (CRRA) in a quasi-linear composite of consumption and labor:

$$U\left(c, (h^j)_{j\in S}\right) = \frac{[c - G((h^j)_{j\in S})]^{1-\sigma} - 1}{1-\sigma},$$

$$G((h^j)_{j\in S}) = \sum_{j\in S} \frac{(h^j)^{\omega_j}}{\omega_j}.$$

with $\sigma, \omega_j > 0$. This specification suggests that labor supplies across sectors are inelastic with respect to wealth. Households maximize their lifetime utility subject to the sequential budget constraint

$$c_t + \sum_{j\in S} i_t^j + \sum_{j\in S} \Phi_j(k_{t+1}^j - k_t^j) + p_t^\tau d_t = p_t^\tau \frac{d_{t+1}}{1+r_t} + \sum_{j\in S} w_t^j h_t^j + \sum_{j\in S} u_t^j k_t^j, \tag{7}$$

where i_t^j, k_t^j, w_t^j, and u_t^j denote, respectively, gross investment, the capital stock, the real wage, and the rental rate of capital in sector j, for $j \in S$. The functions $\Phi_j(\cdot)$, $j \in S$, represent capital adjustment costs and are assumed to be non-negative and convex, and to satisfy the conditions $\Phi_j(0) = \Phi_j'(0) = 0$. They take the form

$$\Phi_j(x) = \frac{\phi_j}{2} x^2.$$

with $\psi, \phi_j > 0$, for $j \in S$. The variable p_t^τ denotes the relative price of the tradable composite goods in terms of final goods, d_t represents the stock of debt in period t, expressed in units of the tradable composite goods, and r_t represents the interest rate on debt held from period t to $t+1$. Consumption, investment, wages, rental rates, debt, and capital adjustment costs are all expressed in units of final goods.

The capital stocks follow the laws of motion

$$k_{t+1}^{j} = (1-\delta)k_t^j + i_t^j, \qquad \text{for} \quad j \in S \tag{8}$$

Using these laws of motion to substitute out i_t^j from the household's budget constraint, and denoting the Lagrange multiplier associated with the resulting budget constraint by $\lambda_t \beta^t$, the first-order optimality conditions with respect to c_t, h_t^j, d_{t+1}, and k_{t+1}^j are, respectively, as follows:

$$U_c(c_t, (h_t^j)_{j\in S}) = \lambda_t, \tag{9}$$

$$-U_{h_t^j}(c_t, (h_t^j)_{j\in S}) = \lambda_t w_t^j, \tag{10}$$

$$\lambda_t p_t^\tau = \beta(1+r_t)E_t \lambda_{t+1} p_{t+1}^\tau, \quad \text{and} \tag{11}$$

$$\lambda_t \left[1 + \Phi_j'(k_{t+1}^j - k_t^j)\right] = \beta E_t \lambda_{t+1} \left[u_{t+1}^j + 1 - \delta + \Phi_j'(k_{t+2}^j - k_{t+1}^j)\right]. \tag{12}$$

For ease of notation, the functions are not substituted with their explicit forms throughout this section. The full equilibrium conditions are provided in Appendix A.4.

4.2 Firms Producing Final Goods

Final goods are produced using nontradable goods and a composite of tradable goods through the technology $B(a_t^\tau, a_t^n)$, where a_t^τ represents the domestic absorption of the tradable composite goods, and a_t^n is the domestic absorption of nontraded goods. The aggregator function $B(\cdot,\cdot)$, assumed to be increasing, concave, and homogeneous of degree 1, is of the CES form [3]:

$$B(a_t^\tau, a_t^n) = \left[\chi_\tau (a_t^\tau)^{1-\frac{1}{\mu_{\tau n}}} + (1-\chi_\tau)(a_t^n)^{1-\frac{1}{\mu_{\tau n}}}\right]^{\frac{1}{1-\frac{1}{\mu_{\tau n}}}},$$

with $\chi_\tau \in (0,1)$ and $\mu_{\tau n} > 0$. These goods are sold to households, which then allocate them to consumption or investment purposes. Firms producing final goods operate competitively, and their profits are given by

$$B(a_t^\tau, a_t^n) - p_t^\tau a_t^\tau - p_t^n a_t^n,$$

[3] Any CES aggregation function in this model takes the Cobb-Douglas form if the associated elasticity of substitution is equal to 1.

where p_t^n is the relative price of nontradable goods in terms of final goods. It follows that the conditions for profit maximization are

$$B_\tau(a_t^\tau, a_t^n) = p_t^\tau; \qquad B_n(a_t^\tau, a_t^n) = p_t^n, \tag{13}$$

where $B_\tau(\cdot,\cdot)$ and $B_n(\cdot,\cdot)$ denote the first derivatives of $B(\cdot,\cdot)$ with respect to a_t^τ and a_t^n, respectively. These conditions determine the domestic demand functions for nontradables and for the tradable composite goods.

4.3 Firms Producing Importable, Exportable, and Tradable Composite Goods

The tradable composite goods are produced using importable and exportable goods as intermediate inputs, through the technology

$$a_t^\tau = A(a_t^m, a_t^x), \tag{14}$$

where a_t^m and a_t^x represent, respectively, the domestic absorptions of importable and exportable goods (both commodities and non commodities). The aggregator function $A(\cdot,\cdot)$, assumed to be increasing, concave, and linearly homogeneous, is of the form

$$A(a_t^m, a_t^x) = \left[\chi_m (a_t^m)^{1-\frac{1}{\mu_{mx}}} + (1-\chi_m)(a_t^x)^{1-\frac{1}{\mu_{mx}}}\right]^{\frac{1}{1-\frac{1}{\mu_{mx}}}},$$

with $\chi_m \in (0,1)$ and $\mu_m > 0$, and the profits of firms producing the tradable composite goods are given by

$$p_t^\tau A(a_t^m, a_t^x) - p_t^m a_t^m - p_t^x a_t^x$$

where p_t^m and p_t^x are, respectively, the overall relative prices of importable and exportable goods in terms of final goods. These firms are also assumed to operate competitively in intermediate and final goods markets. It follows that their profit maximization conditions are

$$p_t^\tau A_1(a_t^m, a_t^x) = p_t^m; \qquad p_t^\tau A_2(a_t^m, a_t^x) = p_t^x, \tag{15}$$

where $A_1(\cdot,\cdot)$ and $A_2(\cdot,\cdot)$ denote the first derivatives of $A(\cdot,\cdot)$ with respect to a_t^m and a_t^x, respectively. These expressions define the domestic demand functions for importable and exportable goods (commodities and non-commodities combined).

The importable and exportable goods are themselves composite goods produced using commodity and non-commodity goods as inputs through the technologies

$$a_t^m = A^m(a_t^{m_c}, a_t^{m_{\bar{c}}}); \qquad a_t^x = A^x(a_t^{x_c}, a_t^{x_{\bar{c}}}), \tag{16}$$

where $a_t^{m_c}$, $a_t^{m_{\bar{c}}}$, $a_t^{x_c}$, and $a_t^{x_{\bar{c}}}$ represent, respectively, the domestic absorptions of commodity-importable, non commodity-importable, commodity-exportable, and non commodity-exportable goods. The aggregator functions $A^m(\cdot,\cdot)$ and $A^x(\cdot,\cdot)$ are increasing, concave, and linearly homogeneous, and of the forms

$$A^m(a_t^{m_c}, a_t^{m_{\bar{c}}}) = \left[\chi_{m,c}(a_t^{m_c})^{1-\frac{1}{\mu_m}} + (1-\chi_{m,c})(a_t^{m_{\bar{c}}})^{1-\frac{1}{\mu_m}}\right]^{\frac{1}{1-\frac{1}{\mu_m}}},$$

$$A^x(a_t^{x_c}, a_t^{x_{\bar{c}}}) = \left[\chi_{x,c}(a_t^{x_c})^{1-\frac{1}{\mu_x}} + (1-\chi_{x,c})(a_t^{x_{\bar{c}}})^{1-\frac{1}{\mu_x}}\right]^{\frac{1}{1-\frac{1}{\mu_x}}},$$

The profits of firms producing the importable and exportable composite goods are given by

$$p_t^m A^m(a_t^{m_c}, a_t^{m_{\bar{c}}}) - p_t^{m_c} a_t^{m_c} - p_t^{m_{\bar{c}}} a_t^{m_{\bar{c}}}; \qquad p_t^x A^x(a_t^{x_c}, a_t^{x_{\bar{c}}}) - p_t^{x_c} a_t^{x_c} - p_t^{x_{\bar{c}}} a_t^{x_{\bar{c}}}$$

where $p_t^{m_c}$, $p_t^{m_{\bar{c}}}$, $p_t^{x_c}$, and $p_t^{x_{\bar{c}}}$ denote, respectively, the relative price of commodity-importable, non commodity-importable, commodity-exportable, and non commodity-exportable goods in terms of final goods. Firms producing these goods are also assumed to behave competitively, and profit maximization implies:

$$\begin{cases} p_t^m A_1^m(a_t^{m_c}, a_t^{m_{\bar{c}}}) = p_t^{m_c}, \\ p_t^m A_2^m(a_t^{m_c}, a_t^{m_{\bar{c}}}) = p_t^{m_{\bar{c}}} \end{cases}; \qquad \begin{cases} p_t^x A_1^x(a_t^{x_c}, a_t^{x_{\bar{c}}}) = p_t^{x_c}, \\ p_t^x A_2^x(a_t^{x_c}, a_t^{x_{\bar{c}}}) = p_t^{x_{\bar{c}}} \end{cases} \tag{17}$$

$A_1^m(\cdot,\cdot)$ and $A_2^m(\cdot,\cdot)$, are, respectively, the first derivatives of $A^m(\cdot,\cdot)$ with respect to $a_t^{m_c}$ and $a_t^{m_{\bar{c}}}$, and $A_1^x(\cdot,\cdot)$ and $A_2^x(\cdot,\cdot)$, are, respectively, the first derivatives of $A^x(\cdot,\cdot)$ with respect to $a_t^{x_c}$ and $a_t^{x_{\bar{c}}}$. These four expressions represent the domestic demand functions for commodity-importable, non commodity-importable, commodity-exportable, and non commodity-exportable goods.

4.4 Firms Producing Intermediate Goods

Goods in sector $j \in S = \{m_c, m_{\bar{c}}, x_c, x_{\bar{c}}, n\}$ are produced with capital and labor through the technologies

$$y_t^j = Z^j F^j(k_t^j, h_t^j), \tag{18}$$

where y_t^j and Z^j denote, respectively, output and a productivity factor in sector j. The production functions $F^j(\cdot,\cdot)$, $j \in S$, are assumed to be increasing in both arguments, concave, and homogeneous of degree 1. Their functional form is Cobb–Douglas and is expressed as follows:

$$F^j(k^j, h^j) = (k^j)^{\alpha_j}(h^j)^{1-\alpha_j}, \quad j \in S,$$

where $\alpha_j \in (0,1)$. The profits of firms producing intermediate goods are given by

$$p_t^j Z^j F^j(k_t^j, h_t^j) - w_t^j h_t^j - u_t^j k_t^j.$$

These firms are assumed to behave competitively in product and factor markets. Then, their first-order profit maximization conditions are

$$p_t^j Z^j F_1^j(k_t^j, h_t^j) = u_t^j\,; \qquad p_t^j Z^j F_2^j(k_t^j, h_t^j) = w_t^j. \tag{19}$$

These efficiency conditions represent the sectoral demand functions for capital and labor. Together with the assumption of linear homogeneity of the production technologies, they imply that firms make zero profits at all times.

4.5 Competitive Equilibrium

In equilibrium, the demand for final goods must equal the supply of this type of goods:

$$c_t + \sum_{j \in S} i_t^j + \sum_{j \in S} \Phi_j(k_{t+1}^j - k_t^j) = B(a_t^\tau, a_t^n). \tag{20}$$

Also, the demand for nontradables must equal the production of nontradables:

$$a_t^n = y_t^n. \tag{21}$$

Imports of commodity and non-commodity goods, denoted respectively m_t^c and $m_t^{\bar{c}}$, are defined as the difference between the domestic absorption and the output of these types of goods:

$$m_t^c = p_t^{m_c}(a_t^{m_c} - y_t^{m_c}), \quad \text{and} \quad m_t^{\bar{c}} = p_t^{m_{\bar{c}}}(a_t^{m_{\bar{c}}} - y_t^{m_{\bar{c}}}) \tag{22}$$

The prices of the importables appear on the right-hand side of these definitions because m_t^c ($m_t^{\bar{c}}$) is expressed in units of final goods, whereas $y_t^{m_c}$ ($y_t^{m_{\bar{c}}}$) and $a_t^{m_c}$ ($a_t^{m_{\bar{c}}}$) are expressed in units of commodity-importable (non commodity-importable) goods. The total imports are given by

$$m_t = m_t^c + m_t^{\bar{c}} \tag{23}$$

Similarly, exports of commodity and non-commodity goods, denoted respectively x_t^c and $x_t^{\bar{c}}$, are defined as the difference between commodity (non-commodity) exportable output, $y_t^{x_c}$ ($y_t^{x_{\bar{c}}}$), and the domestic absorption of commodity (non-commodity) exportables, $a_t^{x_c}$ ($a_t^{x_{\bar{c}}}$):

$$x_t^c = p_t^{x_c}(y_t^{x_c} - a_t^{x_c}), \quad \text{and} \quad x_t^{\bar{c}} = p_t^{x_{\bar{c}}}(y_t^{x_{\bar{c}}} - a_t^{x_{\bar{c}}}) \tag{24}$$

Like imports, exports are measured in terms of final goods. The total exports are given by

$$x_t = x_t^c + x_t^{\bar{c}} \tag{25}$$

Combining Eq (21) to (25), the household's budget constraint, and the definitions of profits in the final- and intermediate-good markets, and taking into account that firms make zero profits at all times yields the following aggregate resource constraint (steps in Appendix A.3):

$$p_t^\tau \frac{d_{t+1}}{1+r_t} = p_t^\tau d_t + m_t - x_t \tag{26}$$

To ensure a stationary equilibrium process for external debt, I follow Schmitt-Grohé and Martín Uribe (2018) and assume that the country interest rate is debt elastic:

$$r_t = r^* + s_t + p(d_{t+1}) \tag{27}$$

where r^* is the risk-free world interest rate, s_t is the global component of the interest-rate spread, and $p(d)$ is the domestic component of the interest-rate spread, with $p(\bar{d}) = 0$ and $p'(\bar{d}) > 0$, for some constant $\bar{d}$.

Although not relevant for the exercises in the paper, I define two measures of terms of trade: the overall terms of trade, defined as the relative price of exportable goods in terms of importable goods, and the commodity terms of trade, defined as the relative price of commodity exportable goods in terms of commodity importable goods:

$$tot_t = \frac{p_t^x}{p_t^m}, \quad \text{and} \quad tot_t^c = \frac{p_t^{x_c}}{p_t^{m_c}} \tag{28}$$

Schmitt-Grohé and Martín Uribe (2018) treat tot_t as exogenously given to explore shocks to tot_t (overall terms-of-trade shocks). However, as argued in Section 2, the overall export-to-import price ratio does not only capture changes in world prices. It therefore cannot reasonably be used to identify exogenous terms-of-trade shocks. tot_t^c is potentially better at capturing these shocks. However, the focus of this paper is not on shocks to tot_t^c, but on what happens when there is a shock to its numerator (commodity export price shock) or its denominator (commodity import price shock). Therefore, as opposed to Schmitt-Grohé and Martín Uribe (2018) who fit an AR(1) process to the ratio tot_t, I fit AR(1) processes to $p_t^{x_c}$ and $p_t^{m_c}$ separately, each of which is exogenously determined in international markets. In addition, the typical emerging or developing country is small in asset markets and therefore takes the evolution of the global component of the interest-rate spread, s_t as exogenously given.

Finally, and in line with the empirical analysis, I assume that $\widehat{p_t}^k, k = x_c$ or m_c, and $\widehat{s}_t$ follow the joint law of motion given in Eq (5), with $\widehat{p_t}^k \equiv \ln(p_t^k / \bar{p}^k)$, $\widehat{s}_t \equiv s_t - \bar{s}$, and $\bar{p}^k$ and $\bar{s}$ denoting the deterministic steady-state values of p_t^k and s_t, respectively.

As in the empirical part, the real exchange rate is defined as the ratio of the foreign consumer price index to the domestic consumer price index. Formally,

$$RER_t = \frac{\mathcal{E}_t P_t^*}{P_t}, \tag{29}$$

where $\mathcal{E}_t$ represents the nominal exchange rate, defined as the price of one unit of foreign currency in terms of the domestic currency; P_t^* is the foreign price of consumption, and P_t is the domestic price of consumption. By dividing both the numerator and denominator by the domestic-currency price of the tradable composite goods, denoted P_t^τ, we obtain $RER_t = \left(\frac{\mathcal{E}_t P_t^*}{P_t^\tau}\right) / \left(\frac{P_t}{P_t^\tau}\right)$. Assuming the law of one price holds for all importable and exportable goods, and that the aggregator functions for the importable, exportable, and tradable composite goods, that is $A_m(\cdot,\cdot)$, $A_x(\cdot,\cdot)$, and $A(\cdot,\cdot)$, are common across countries, the law of one price must also hold for the tradable composite goods. This implies that $\mathcal{E}_t P_t^{\tau} = P_t^\tau$, where P_t^τ is the foreign price of the tradable composite goods. This yields $RER_t = \left(\frac{P_t^*}{P_t^\tau}\right) / \left(\frac{P_t}{P_t^\tau}\right)$. Next, assuming that the commodity-price shocks relevant to the small open economy do not affect the relative price of the tradable composite goods in terms of consumption goods in the rest of the world, it follows that $\frac{P_t^*}{P_t^\tau}$ is constant. Without loss of generality, $\frac{P_t^*}{P_t^\tau}$ is normalized to 1. Finally, noting that $p_t^\tau \equiv \frac{P_t^\tau}{P_t}$, we have

$$RER_t = p_t^\tau. \tag{30}$$

In other words, the real exchange rate equals the relative price of the tradable composite goods in terms of final goods.

A competitive equilibrium in the MXN-C model is a set of 54 processes k^j_{t+1}, i^j_t, h^j_t, w^j_t, u^j_t, a^j_t, y^j_t for $j \in S$; c_t, λ_t, p^τ_t, RER_t, r_t, a^τ_t, p^m_t, p^x_t, $p^{m_{\bar{c}}}_t$, $p^{x_{\bar{c}}}_t$, a^n_t, p^n_t, m^c_t, $m^{\bar{c}}_t$, x^c_t, $x^{\bar{c}}_t$, m_t, x_t, and d_{t+1}, satisfying Equations (7) to (30), given initial conditions k^j_0 for $j \in S$, and d_0, and the stochastic process for p^k_t, $k = x_c$ or m_c, and s_t.

4.6 Observables

In the MXN-C model, consumption (c_t), GDP (y_t), investment (i_t), and the trade balance (tb_t) are all expressed in units of final (consumption) goods. The latter three variables are given by

$$y_t = \sum_{j \in S} p^j_t y^j_t, \qquad i_t = \sum_{j \in S} i^j_t, \qquad tb_t = x_t - m_t. \tag{31}$$

However, data used in the empirical SVAR model are not expressed in terms of final consumption goods. For a sensible comparison of the model predictions with data, the theoretical and empirical variables must be expressed in the same units. In the SVAR model, GDP, consumption, investment, and the trade balance are deflated by a Paasche GDP deflator. Following Schmitt-Grohé and Martín Uribe (2018), I derive the corresponding theoretical counterparts in what follows. In the MXN-C model, GDP at current prices is given by

$$\sum_{j \in S} P^j_t y^j_t$$

where P^j_t is the nominal price of goods j in period t. The data source (WDI) uses a Paasche index for the GDP deflator, defined as the ratio of current-price to constant-price GDP. Formally, the GDP deflator in period t is given by

$$\frac{\sum_{j \in S} P^j_t y^j_t}{\sum_{j \in S} P^j_0 y^j_t}$$

where $t = 0$ indicates the base year. Real GDP is given by dividing nominal GDP by the GDP deflator, that is,

$$\sum_{j \in S} P^j_0 y^j_t$$

The nominal prices in the base year, P^j_0, and all other nominal prices in period 0 are indices without a real unit. This means I can set one nominal base price arbitrarily, without loss of generality. Thus I set the nominal price of consumption in period 0 equal to 1, $P_0 = 1$. This implies that $P^j_0 = p^j_0$ for $j \in S$. It follows that real output in period t is

$$\sum_{j \in S} p^j_0 y^j_t$$

Finally, I assume that in the base period the economy was in the deterministic steady state, so that $p_0^j = p^j$ for $j \in S$. Then, the theoretical counterpart of the observed measure of real GDP, denoted by y_t^o, is given by

$$y_t^0 = \sum_{j \in S} p^j y_t^j$$

Similarly, the theoretical counterpart of real consumption is the ratio of nominal consumption, $P_t c_t$, to the GDP deflator, that is

$$c_t^o \equiv P_t c_t \frac{\sum_{j \in S} P_0^j y_t^j}{\sum_{j \in S} P_j^j y_t^j}.$$

Recalling that $p_t^j \equiv P_t^j / P_t$ and that $P_0^j = p^j$ for $i \in S$, we can write the theoretical counterpart of observed real consumption as

$$c_t^o \equiv c_t \frac{\sum_{j \in S} p^j y_t^j}{\sum_{j \in S} p_j^j y_t^j}.$$

The theoretical counterparts of observed real investment and the trade balance are derived in the same way:

$$i_t^o \equiv i_t \frac{\sum_{j \in S} p^j y_t^j}{\sum_{j \in S} p_j^j y_t^j}, \quad \text{and} \quad tb_t^o \equiv tb_t \frac{\sum_{j \in S} p^j y_t^j}{\sum_{j \in S} p_j^j y_t^j}.$$

The variables y_t^o, c_t^o, i_t^o, and tb_t^o are those used when comparing the predictions of the theoretical model to the data, rather than the corresponding measures in terms of final goods, y_t, c_t, i_t, and tb_t.

4.7 Calibration

Table (4) summarizes the calibration and estimation results for the parameters appearing in the steady-state equilibrium conditions of the MXN-C model (Appendix A.5). Steady-state values are those without a time subscript. The equilibrium conditions evaluated at the steady state form a system of 54 equations in 84 unknowns, that is, the 54 endogenous variables specified in the definition of the competitive equilibrium, and 30 structural parameters, namely, Z^j, ω^j, α^j for $j \in S$; δ, β, $\chi_{m,c}$, $\chi_{x,c}$, χ_m, χ_τ, μ_m, μ_x, μ_{mx}, $\mu_{\tau n}$, $r^* + \bar{s}$, $\bar{d}$, σ, and $\bar{p}^k$, $k \in \{x_c, m_c\}$. Therefore, we need 30 calibration restrictions, which I discuss next.

The parameters $\delta = 0.1$, $\alpha_n = 0.25$, $r^* + \bar{s} = 0.11$, $\mu_{mx} = 1$, $\mu_{\tau n} = 0.5$, and $\sigma = 2$ are taken from Schmitt-Grohé and Martín Uribe (2018) [*6 calibration restrictions*]. The capital shares are assumed to be the same in the commodity and non-commodity importable and exportable sectors ($\alpha^{x_c} = \alpha^{x_{\bar{c}}} = \alpha^{m_c} = \alpha^{m_{\bar{c}}} = 0.35$) [*4 calibration restrictions*]. This is also based on Schmitt-Grohé and Martín Uribe (2018) who assume a value of 0.35 for the capital shares in the importable and exportable sectors. I assume a Cobb-Douglas form for the aggregator functions $A^m(\cdot, \cdot)$ and $A^x(\cdot, \cdot)$, which implies that the elasticities of substitution between commodity and non-commodity inputs in the importable and exportable sectors (μ_m and μ_x) are 1 [*2 calibration restrictions*]. As in Schmitt-Grohé and Martín Uribe (2018), I set $w^j = 1.455$, for $j \in S$, which implies a sectoral Frisch elasticity of labor supply of 2.2 [*5 calibration restrictions*]. The productivity factors $Z^j, j \in S$ are normalized to 1 [*5 calibration restrictions*].

Next, I impose a set of moment restrictions based on data on the pool of 28 countries, which help calculate $\chi_{m,c}$, $\chi_{x,c}$, χ_m, χ_τ, and $\bar{d}$. From UNCTAD's 2021 State of Commodity Dependence report, the average share of commodity exports in GDP for my set of countries, which I denote s_{x_c}, is about 10%. So I set $s_{x_c} = \frac{x^c}{y} = 0.1$ [*1 calibration restriction*]. From the same report, the average share of commodity exports in total merchandise exports is about 56.17%, which implies that the average share of non-commodity exports in total merchandise exports is 43.83%. It follows that the average share of non-commodity exports in GDP is $s_{x_{\bar{c}}} = \frac{x^{\bar{c}}}{y} = (43.83/56.17) \times 0.1 = 0.078$ [*1 calibration restriction*]. I follow the usual practice of approximating the share of non-tradable output in total output (s_{y_n}) by the share of services value added in GDP. Using data from the World Development Indicators (WDI) for the 28 countries, I find a value of about 50% on average. So I set $s_{y_n} = \frac{p^n y^n}{y} = 0.5$ [*1 calibration restriction*]. That value is consistent with Schmitt-Grohé and Martín Uribe (2018). Next, the average trade balance-to-GDP ratio in my sample is $s_{tb} = \frac{x-m}{y} = -0.047$ [*1 calibration restriction*]. We can calculate the share of total imports in total output as $s_m = (s_{x_c} + s_{x_{\bar{c}}}) - s_{tb} = 0.1 + 0.078 + 0.047 = 0.225$. From the UNCTAD report, the average share of commodity imports in total merchandise imports is about 32.47% for my set of countries. It follows that the share of commodity imports in total output is $s_{m_c} = \frac{m^c}{y} = 0.3247 \times 0.225 = 0.073$ [*1 calibration restriction*]. This implies that the share of non-commodity imports in total output is $s_{m_{\bar{c}}} = \frac{m^{\bar{c}}}{y} = s_m - s_{m_c} = 0.152$ [*1 calibration restriction*]. The initial prices p^{x_c} and p^{m_c}, along with prices in the other 3 sectors are derived using a numerical minimization routine given the other calibrated parameters, the moment restrictions, and the steady-state equilibrium conditions. Their values are $p^{x_c} \approx 0.2867$ and $p^{m_c} \approx 0.2565$ [*2 calibration restrictions*]. This completes our set of 30 calibration restrictions.

The following additional calculations are not part of the calibration restrictions. To calculate $\chi_{m,c}$, $\chi_{x,c}$, χ_m, χ_τ, and $\bar{d}$, we also need the shares of the outputs of sectors $x_c, x_{\bar{c}}, m_c$, and $m_{\bar{c}}$ in total ouput, denoted $s_{y_{x,c}}, s_{y_{x,\bar{c}}}, s_{y_{m,c}}$, and $s_{y_{m,\bar{c}}}$, respectively (see Appendix A.5). These values are obtained by solving a system of four equations as follows: (1) I assume that the overall exportable and importable sectors are about the same size, that is, $\frac{s_{y_{m,c}}+s_{y_{m,\bar{c}}}}{s_{y_{x,c}}+s_{y_{x,\bar{c}}}} = 1$. This is based on Schmitt-Grohé and Martín Uribe (2018) who estimate that the exportable and importable sectors in emerging and poor countries are about the same size using UNCTAD data. (2) $s_{y_{m,\bar{c}}} = 1 - s_{y_{x,c}} - s_{y_{x,\bar{c}}} - s_{y_{m,c}} - s_{y_n}$. (3) I assume that the relative size of the non-commodity exportable sector with respect to the commodity exportable sector is given by the ratio of non-commodity exports to commodity exports. That is, $\frac{s_{y_{x,\tilde{c}}}}{s_{y_{x,c}}} = \frac{s_{x_{\tilde{c}}}}{s_{x_c}} = 0.78$. (4) Finally, and similarly to (3), the relative size of the non-commodity importable sector with respect to the commodity importable sector is given by the ratio of non-commodity imports to commodity imports. That is, $\frac{s_{y_{m,\tilde{c}}}}{s_{y_{m,c}}} = \frac{s_{m_{\tilde{c}}}}{s_{m_c}} = 2.08$. Solving the system of four equations yields:

$$\begin{cases} s_{y_{m,c}} \approx 0.0812; & s_{y_{m,\tilde{c}}} \approx 0.1689 \\ s_{y_{x,c}} \approx 0.1404; & s_{y_{x,\tilde{c}}} \approx 0.1095 \end{cases} \tag{32}$$

The parameters a_{ij}, π_{ij} for $i, j = 1, 2$, ϕ_j, for $j \in S$, and ψ do not appear in the steady-state equilibrium conditions but play a role in the equilibrium dynamics. Their estimation follows closely the methods laid out in Schmitt-Grohé and Martín Uribe (2018). I assign values to a_{ij} and π_{ij} country by country using the SVAR estimates. The capital adjustment cost parameters, ϕ_j and the parameter ψ governing the debt elasticity of the country premium are estimated using a partial information method. Specifically, these parameters are set to minimize a weighted difference between the impulse responses to commodity import/export price shocks and interest-rate-spread shocks implied by the SVAR and MXN-C models. I consider the first 5 years of each impulse response function, and the weights are the reciprocal of the width of the 66% confidence intervals associated with the SVAR impulse responses. Formally, let $\Theta = [(\phi_j)_{j \in S}, \psi]$. Then Θ is the solution to the problem:

$$\min_{\Theta} \sum_{h=p^k,s} \sum_{i=0}^{4} \sum_{j=y^o,c^o,i^o,tb^o,RER} \frac{\left| IR^{SVAR}_{hij} - IR^{MXN-C}_{hij}(\Theta) \right|}{\Delta_{hij}} \tag{33}$$

where IR^{SVAR}_{hij} and $IR^{MXN-C}_{hij}(\Theta)$ denote the impulse response of variable j, i periods after a shock h implied by the SVAR and MXN-C models, respectively, and Δ_{hij} is the width of the 66% confidence band associated with IR^{SVAR}_{hij}. The parameters are estimated country by country and presented in Appendix A.2.

Table 4: Calibration of the MXN-C Model

Calibrated Parameters							
σ	$\omega^j,\ j \in S$	δ	α_n	$\alpha^{x_c}, \alpha^{x_{\tilde{c}}}, \alpha^{m_c}, \alpha^{m_{\tilde{c}}}$	$r^* + \bar{s}$	μ_x, μ_m	μ_{mx}
2	1.455	0.1	0.25	0.35	0.11	1	1
Moment Restrictions							
$s_{x_c} = x^c/y$	$s_{x_{\tilde{c}}} = x^{\tilde{c}}/y$	$s_{y_n} = \frac{p^n y^n}{y}$	$s_{tb} = \frac{x-m}{y}$	$s_{m_c} = m^c/y$	$s_{m_{\tilde{c}}} = m^{\tilde{c}}/y$		
0.1	0.078	0.5	-0.047	0.073	0.152		
Implied Structural Parameter Values							
β	$\chi_{m,c}$	$\chi_{x,c}$	χ_m	χ_τ	$\bar{d}$		
$1/(1 + r^* + \bar{s})$	0.3246	0.5619	0.8685	0.3452	-0.0086		

4.8 The Importance of Commodity-Price Shocks: Theoretical Predictions

In this section, I present the MXN-C model's predictions on the importance of commodity-price shocks and compare them with those of the SVAR model. Figures (4) and (5) present the median impulse responses implied by the SVAR and MXN-C models (See online Appendix for country-specific comparisons). The model appears to fit the data poorly. Overall, the impulse responses appear more muted in the MXN-C model than in the SVAR model. Table (5) below shows the median variance shares explained by the shocks. Focusing on the five business-cycle variables of interest, the average of the median variance shares explained by commodity export-price shocks in the MXN-C model is about 2%, compared to 26% in the SVAR model. For commodity import-price shocks, this average is about 3% in the MXN-C model, compared to 30% in the SVAR model. These differences are huge, suggesting that in the aggregate, there is a high level of mismatch between theory and empirics when it comes to quantifying the importance of commodity-price shocks in EMDEs.

Finally, I also examine the variance shares at the country level. Figures (6) and (7) plot the variance shares implied by the SVAR model against the corresponding variance shares implied by the MXN-C model. In all four figures, the cloud of points fails to trace out the 45° line. For the commodity import-price shocks, the correlations between the variance shares predicted by SVAR and MXN-C models are -0.52, -0.36, -0.28, 0.1, and -0.16 for output, consumption, investment, the real exchange rate, and the trade balance, respectively. This suggests a strong disconnect between the empirical and theoretical models. For the commodity export-price shocks, these correlations are 0, -0.12, 0.13, 0.18, and 0, respectively. Here too, the SVAR and MXN-C models are disconnected, although to a lesser extent than in the case of commodity import price shocks.

In summary, according to the empirical SVAR model, commodity price shocks play an important role in explaining business-cycle fluctuations in EMDEs. However, the predictions of the MXN-C model suggest otherwise. Not only do these shocks play a negligible role average in the MNX-C model, but its predictions are also disconnected from those of the SVAR model at the country level.

Table 5: Median Variance Shares Explained by Commodity-Price Shocks: MXN-C vs. SVAR

	Commodity export-price shocks							Commodity import-price shocks						
Country	$\widehat{p}^{x_c}$	$\widehat{s}$	$\widehat{tb}$	$\widehat{y}$	$\widehat{c}$	$\widehat{i}$	$\widehat{rer}$	$\widehat{p}^{m_c}$	$\widehat{s}$	$\widehat{tb}$	$\widehat{y}$	$\widehat{c}$	$\widehat{i}$	$\widehat{rer}$
MXN-C Model	90.48	4.40	4.31	1.32	0.54	5.93	0	98.73	6.76	11.37	3.47	1.06	0.09	0
SVAR Model	90.48	4.40	20.65	33.55	25.72	24.49	26.70	98.73	6.76	25.42	29.49	29.72	33.37	33.16

Notes: Shares are expressed in percent.

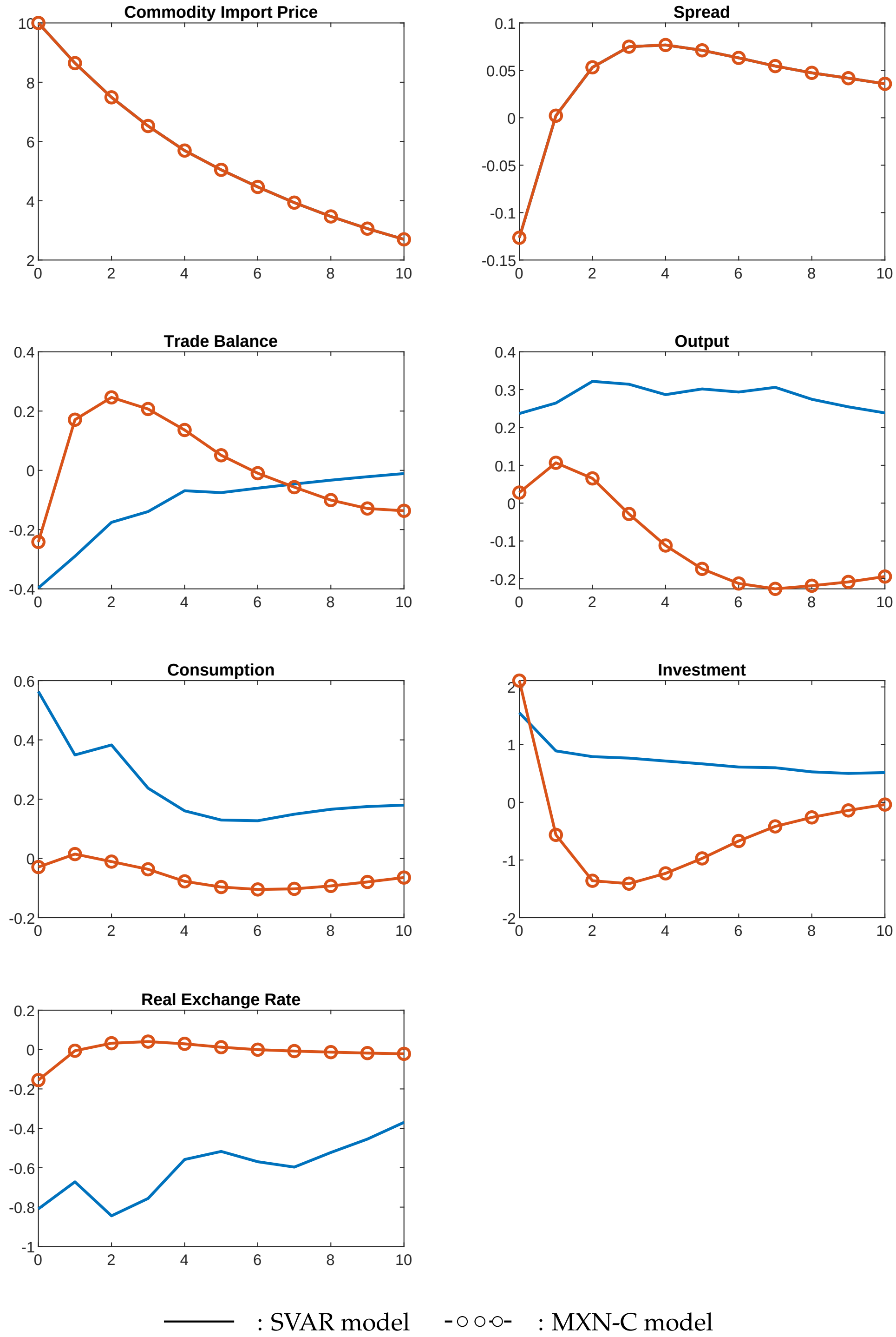

Figure 4: Comparison of the impulse responses to commodity import-price shocks implied by the SVAR and MXN-C models. The y-axes represent percent deviations from trend. For the trade balance, the y-axis represents the percent deviation from GDP trend. The x-axes represent years after the shock

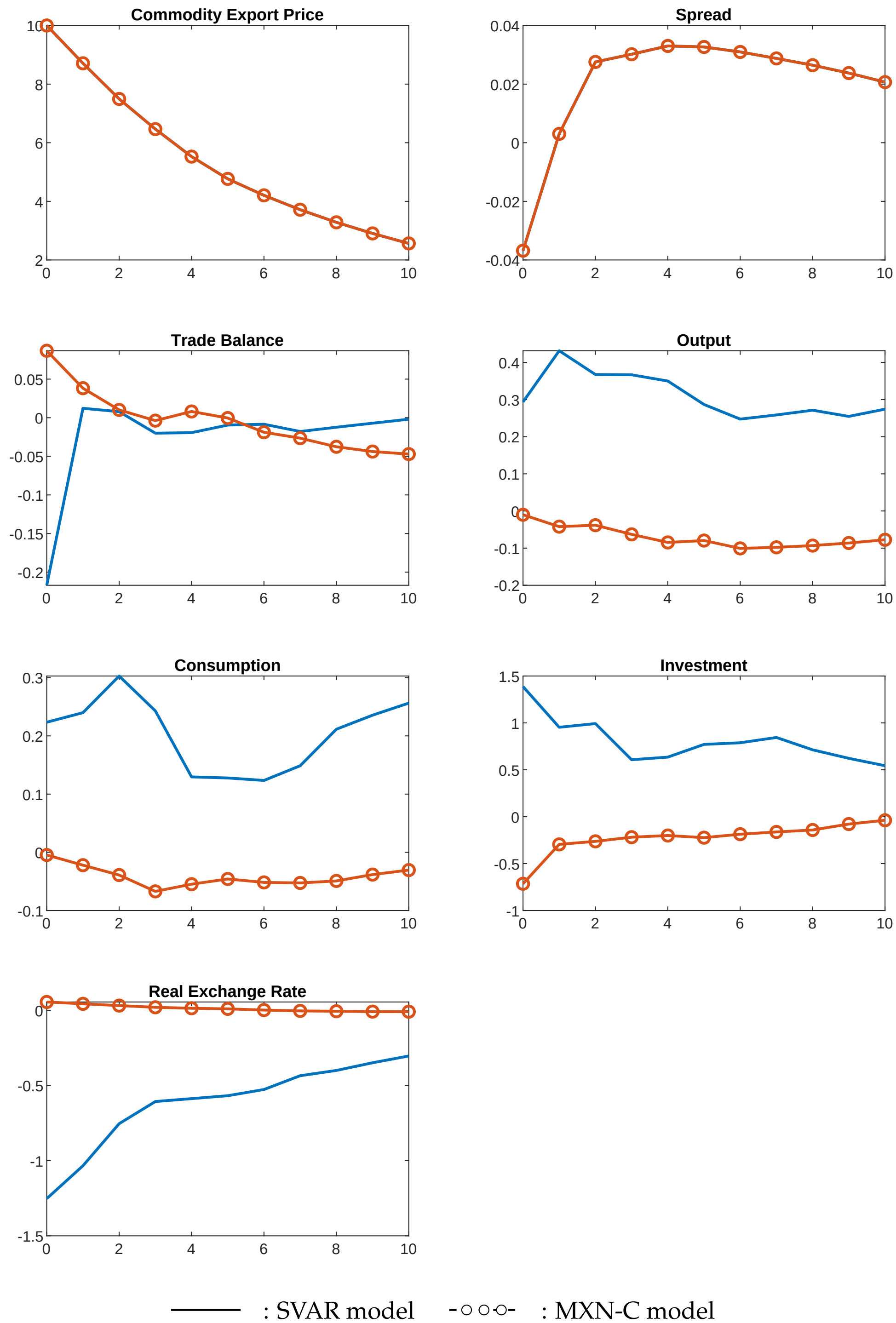

Figure 5: Comparison of the impulse responses to commodity export-price shocks implied by the SVAR and MXN-C models. The y-axes represent percent deviations from trend. For the trade balance, the y-axis represents the percent deviation from GDP trend. The x-axes represent years after the shock

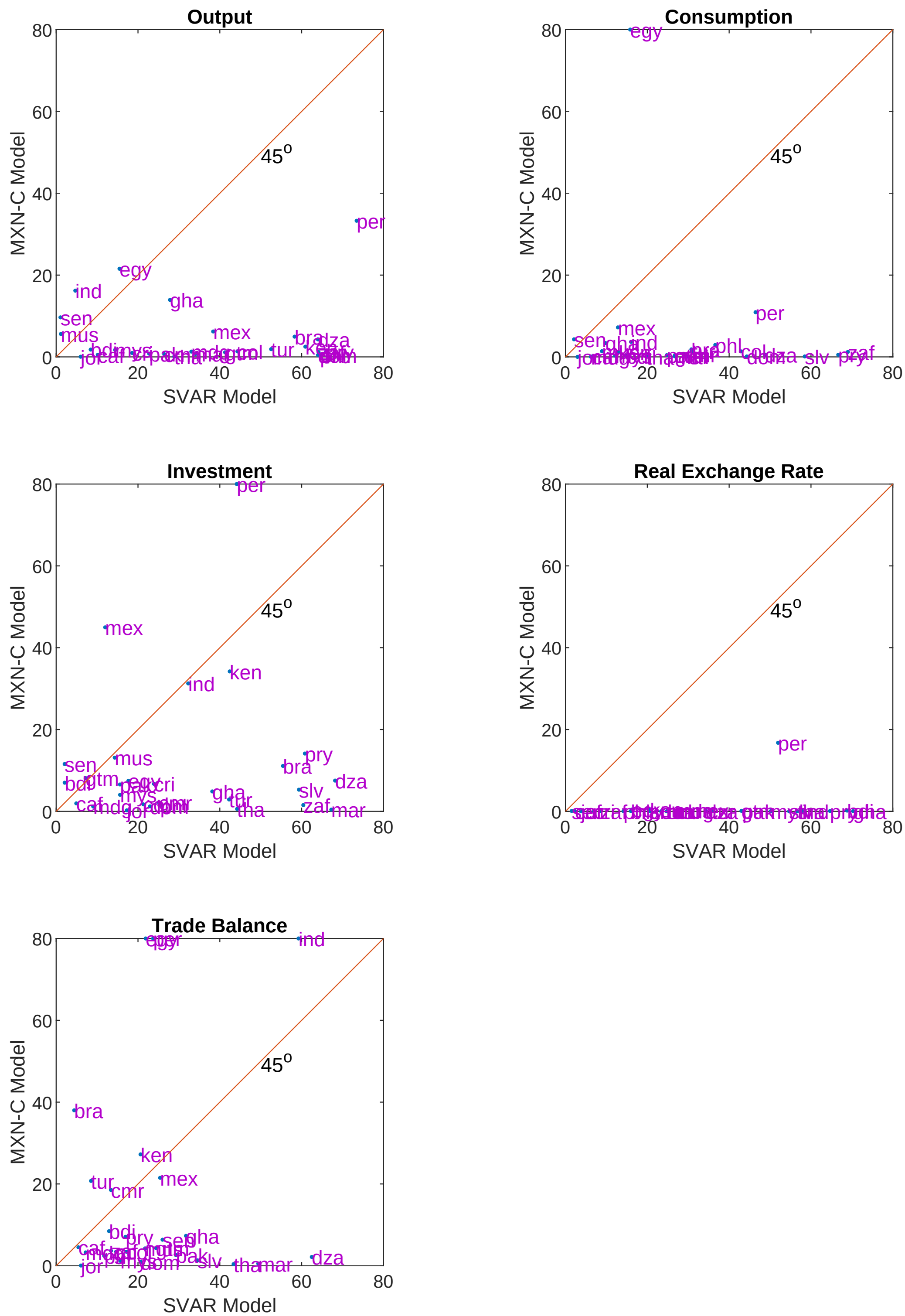

Figure 6: Shares of variances explained by commodity export-price shocks: SVAR vs. MXN-C. Shares are expressed in percent. The correlations between the variance shares are 0, -0.12, 0.13, 0.18, and 0 for output, consumption, investment, the real exchange rate, and the trade balance, respectively. Like in the baseline model in Schmitt-Grohé and Martín Uribe (2018), some of the variance shares predicted by the MXN-C model are larger than 100%, potentially owing to the complexity of the model. These are the data points appearing on the 80% limit.

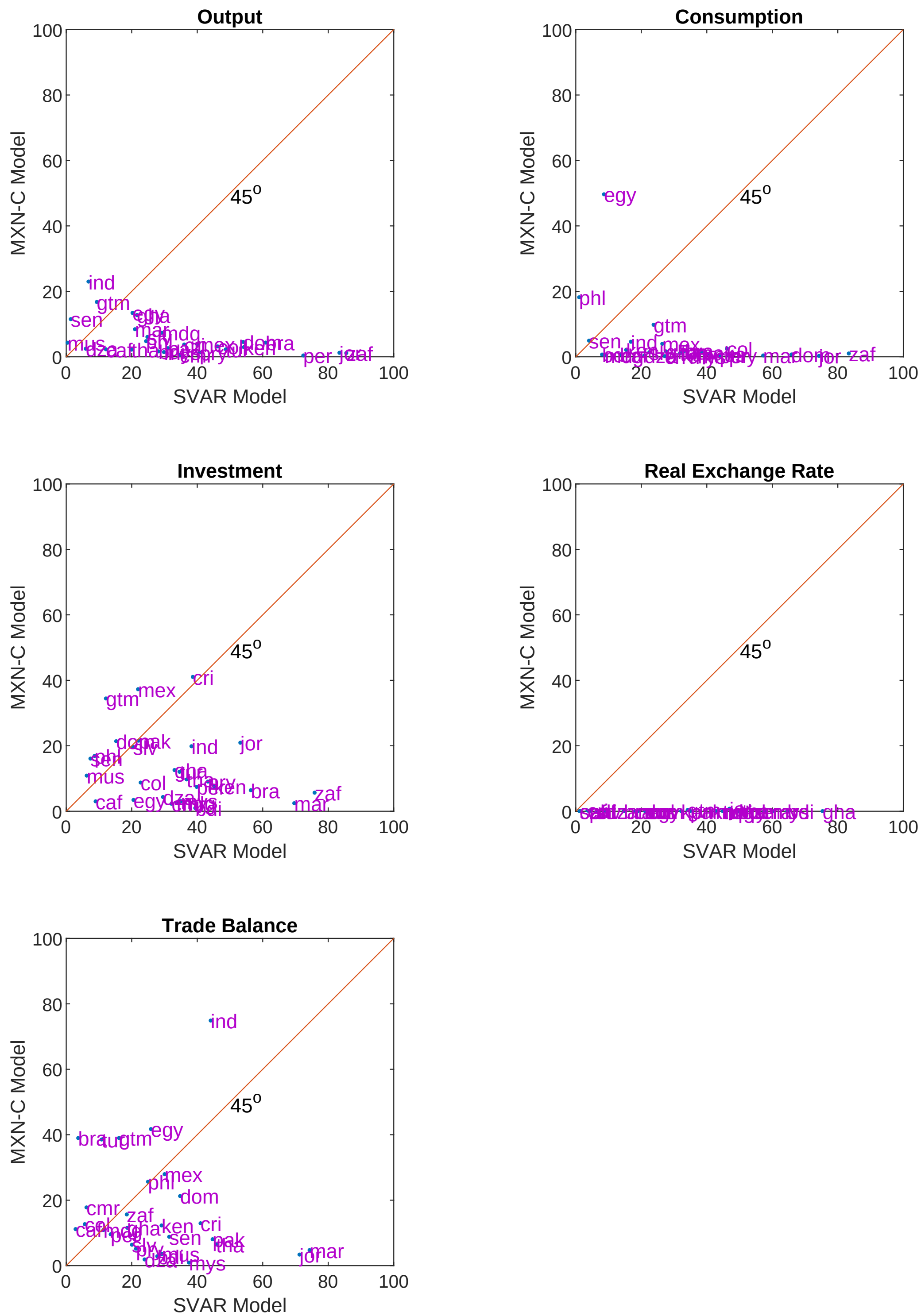

Figure 7: Shares of variances explained by commodity import-price shocks: SVAR vs. MXN-C. Shares are expressed in percent. The correlations between the variance shares are -0.52, -0.36, -0.28, 0.1, and -0.16 for output, consumption, investment, the real exchange rate, and the trade balance, respectively.

5 Concluding Remarks

This paper quantifies the importance of commodity import and export price shocks as a source of business-cycle fluctuations in emerging market and developing economies (EMDEs). Using a sample of 28 EMDEs and data from 1980 to 2019, I estimate an SVAR model and find that these shocks explain an important fraction (26-30%) of aggregate fluctuations in these countries. Building on Schmitt-Grohé and Martín Uribe (2018), I then develop a 5-sector RBC model that explicitly introduces commodity prices. Using this model, I calculate the same variance shares as in the SVAR model. The RBC model predicts that commodity import and export price shocks explain on average only a small fraction (2-3%) of aggregate fluctuations in the selected countries. In addition, a country-by-country comparison of the variance shares predicted by SVAR and RBC models shows that there is a disconnect between these models when it comes to measuring the importance of commodity-price shocks in EMDEs.

The empirical findings confirm the conventional view that world price shocks play an important role in generating business cycles in developing countries. However, the stark differences between empirical and theoretical results in this paper underscore the limitations of current MXN models in capturing the business-cycle dynamics of small open developing economies in response to world price shocks. Future research will aim to bridge the gap between MXN models and the data by examining several key dimensions, including calibration methods and modeling assumptions.

References

Akıncı, Özge (2013). "Global financial conditions, country spreads and macroeconomic fluctuations in emerging countries". In: *Journal of International Economics* 91.2, pp. 358–371. DOI: https://doi.org/10.1016/j.jinteco.2013.07.005.

Broda, Christian (2004). "Terms of trade and exchange rate regimes in developing countries". In: *Journal of International economics* 63.1, pp. 31–58. DOI: https://doi.org/10.1016/S0022-1996(03)00043-6.

Chen, Yu-chin and Kenneth Rogoff (2003). "Commodity currencies". In: *Journal of international Economics* 60.1, pp. 133–160.

Di Pace, Federico, Luciana Juvenal, and Ivan Petrella (2020). "Terms-of-trade shocks are not all alike". In.

Fernández-Villaverde, Jesús et al. (2011). "Risk matters: The real effects of volatility shocks". In: *American Economic Review* 101.6, pp. 2530–2561. DOI: 10.1257/aer.101.6.2530.

Garcia-Cicco, Javier, Roberto Pancrazi, and Martin Uribe (2010). "Real business cycles in emerging countries?" In: *American Economic Review* 100.5, pp. 2510–2531. DOI: 10.1257/aer.100.5.2510.

Gruss, Bertrand and Suhaib Kebhaj (2019). *Commodity terms of trade: A new database*. International Monetary Fund. URL: https://www.imf.org/en/Publications/WP/Issues/2019/01/24/Commodity-Terms-of-Trade-A-New-Database-46522.

Kose, M Ayhan (2002). "Explaining business cycles in small open economies:'How much do world prices matter?'" In: *Journal of International Economics* 56.2, pp. 299–327. DOI: https://doi.org/10.1016/S0022-1996(01)00120-9.

Mendoza, Enrique G (1995). "The terms of trade, the real exchange rate, and economic fluctuations". In: *International Economic Review*, pp. 101–137. DOI: https://doi.org/10.2307/2527429.

Schmitt-Grohé, Stephanie and Martín Uribe (2018). "How Important are Terms-Of-Trade Shocks?" In: *International Economic Review* 59.1, pp. 85–111. DOI: https://doi.org/10.1111/iere.12263.

A Appendix

A.1 Data Sources and Transformations

Data on commodity prices come from the International Monetary Fund's Commodity Terms of Trade database put together by Gruss and Kebhaj (2019). All other data come from the World Bank's World Development Indicators (September 2023 vintage) and are described in the table below.

Variable	Description	Code
NBTT	Net barter terms of trade index (2000=100)	TT.PRI.MRCH.XD.WD
M	Imports of goods and services (% of GDP)	NE. IMP. GNFS. ZS
X	Exports of goods and services (% of GDP)	NE. EXP. GNFS. ZS
Y	GDP per capita in constant LCU	NY.GDP. PCAP. KN
C	HHs & NPISHs final cons. exp. (% of GDP)	NE.CON.PRVT.ZS
I	Gross capital formation (% of GDP)	NE. GDI. TOTL. ZS
NER	Official exchange rate (LCU/US$, period avg.)	NY.GDP. PCAP. KN
P	Consumer Price Index (2010 = 100)	FP.CPI.TOTL

The base year for the consumer price indices (CPIs) is 2010. For consistency, the commodity price indices, the net barter terms of trade, the commodity terms of trade, and the real exchange rate are also adjusted to a 2010 base year. The real commodity price indices are then calculated by dividing the base-2010 commodity price indices by the corresponding CPIs.

The data covers the period from 1980 to 2019. The trade balance is calculated as $TB = X - M$. All variables (except the trade balance) are log-quadratically detrended. The trade balance takes negative values, so it cannot be log-transformed. Instead, it is first divided by the quadratic trend component of output (Y) and the resulting ratio is quadratically detrended.

Table 6: List of Commodities

Category	Commodity	Commodity - Detailed Price Source
Agricultural raw materials	Cotton	Cotton, Cotton Outlook 'A Index', Middling 1-3/32 inch staple, CIF Liverpool, US cents per pound
	Hard logs	Hard Logs, Best quality Malaysian meranti, import price Japan, US$ per cubic meter
	Hard sawnwood	Hard Sawnwood, Dark Red Meranti, select and better quality, C&F U.K port, US$ per cubic meter
	Hides	Hides, Heavy native steers, over 53 pounds, wholesale dealer's price, US, Chicago, fob Shipping Point, US cents per pound
	Natural rubber	Rubber, Singapore Commodity Exchange, No. 3 Rubber Smoked Sheets, 1st contract, US cents per pound
	Soft logs	Soft Logs, Average Export price from the U.S. for Douglas Fir, US$ per cubic meter
	Soft sawnwood	Soft Sawnwood, average export price of Douglas Fir, U.S. Price, US$ per cubic meter
	Wool	Wool Index, 2005 = 100, includes Coarse and Fine Wool Price Indices
Energy	Coal	Coal, Australian thermal coal, 12,000- btu/pound, less than 1% sulfur, 14% ash, FOB Newcastle/Port Kembla, US$ per metric ton
	Crude oil	Crude Oil (petroleum), Price index, 2005 = 100, simple average of three spot prices; Dated Brent, West Texas Intermediate, and the Dubai Fateh
	Natural gas	Natural Gas Price Index, 2005 = 100, includes European, Japanese, and American Natural Gas Price Indices
Food and Beverages	Bananas	Bananas, Central American and Ecuador, FOB U.S. Ports, US$ per metric ton
	Barley	Barley, Canadian no.1 Western Barley, spot price, US$ per metric ton
	Beef	Beef, Australian and New Zealand 85% lean fores, CIF U.S. import price, US cents per pound
	Chicken	Poultry (chicken), Whole bird spot price, Ready-to-cook, whole, iced, Georgia docks, US cents per pound
	Cocoa	Cocoa beans, International Cocoa Organization cash price, CIF US and European ports, US$ per metric ton
	Coffee	Coffee Price Index, 2005 = 100, includes Other Mild Arabicas and Robusta
	Corn	Maize (corn), U.S. No.2 Yellow, FOB Gulf of Mexico, U.S. price, US$ per metric ton
	Fish	Fish (salmon), Farm Bred Norwegian Salmon, export price, US$ per kilogram
	Fish meal	Fishmeal, Peru Fish meal/pellets 65% protein, CIF, US$ per metric ton
	Groundnuts	Groundnuts (peanuts), 40/50 (40 to 50 count per ounce), cif Argentina, US$ per metric ton
	Lamb	Lamb, frozen carcass Smithfield London, US cents per pound
	Olive oil	Olive Oil, extra virgin less than 1% free fatty acid, ex-tanker price U.K., US$ per metric ton
	Oranges	Oranges, miscellaneous oranges CIF French import price, US$ per metric ton
	Palm oil	Palm oil, Malaysia Palm Oil Futures (first contract forward) 4-5 percent FFA, US$ per metric ton
	Pork	Swine (pork), 51-52% lean Hogs, U.S. price, US cents per pound.
	Rapeseed oil	Rapeseed oil, crude, fob Rotterdam, US$ per metric ton
	Rice	Rice, 5 percent broken milled white rice, Thailand nominal price quote, US$ per metric ton
	Shrimp	Shrimp, No.1 shell-on headless, 26-30 count per pound, Mexican origin, New York port, US$ per kilogram
	Soybean meal	Soybean Meal, Chicago Soybean Meal Futures (first contract forward) Minimum 48 percent protein, US$ per metric ton
	Soybean oil	Soybean Oil, Chicago Soybean Oil Futures (first contract forward) exchange approved grades, US$ per metric ton
	Soybeans	Soybeans, U.S. soybeans, Chicago Soybean futures contract (first contract forward) No. 2 yellow and par, US$ per metric ton
	Sugar	Sugar, Free Market, Coffee Sugar and Cocoa Exchange (CSCE) contract no.11 nearest future position, US cents per pound
	Sunflower seed oil	Sunflower oil, Sunflower Oil, US export price from Gulf of Mexico, US$ per metric ton
	Tea	Tea, Mombasa, Kenya, Auction Price, US cents per kilogram, From July 1998,Kenya auctions, Best Pekoe Fannings. Prior, London auctions, c.i.f. U.K. warehouses
	Wheat	Wheat, No.1 Hard Red Winter, ordinary protein, Kansas City, US$ per metric ton
Metals	Aluminum	Aluminum, 99.5% minimum purity, LME spot price, CIF UK ports, US$ per metric ton
	Copper	Copper, grade A cathode, LME spot price, CIF European ports, US$ per metric ton
	Gold	Gold (UK), 99.5% fine, London afternoon fixing, Average of daily rates
	Iron ore	China import Iron Ore Fines 62% FE spot (CFR Tianjin port), US dollars per metric ton
	Lead	Lead, 99.97% pure, LME spot price, CIF European Ports, US$ per metric ton
	Nickel	Nickel, melting grade, LME spot price, CIF European ports, US$ per metric ton
	Tin	Tin, standard grade, LME spot price, US$ per metric ton
	Uranium	Uranium, NUEXCO, Restricted Price, Nuexco exchange spot, US$ per pound
	Zinc	Zinc, high grade 98% pure, US$ per metric ton

Source: Gruss and Kebhaj (2019)

A.2 Additional Structural Parameter Estimation

Table 7: Country-Specific Estimates of the Capital Adjustment Cost Parameters and the Debt Elasticity of the Interest Rate corresponding to the Commodity Import-Price shocks

Country	ϕ_{m_c}	$\phi_{m_{\bar{c}}}$	ϕ_{x_c}	$\phi_{x_{\bar{c}}}$	ϕ_n	ψ
Algeria	28.63	1.92	78.97	64.04	76.89	0.99
Brazil	79.49	39.21	50.10	57.66	74.61	0.98
Burundi	77.21	16.50	71.68	34.36	79.00	0.99
Cameroon	66.45	21.40	60.57	37.34	76.56	0.89
Central Afr. Rep.	78.16	79.91	23.28	27.52	77.30	0.97
Colombia	72.95	77.59	40.78	66.70	75.29	0.82
Costa Rica	74.57	79.68	76.86	49.92	73.83	0.95
Dominican Rep.	73.22	15.66	72.96	72.06	68.51	0.98
Egypt	63.08	7.82	78.15	75.57	75.00	0.11
El Salvador	41.95	71.15	74.60	65.80	76.99	0.89
Ghana	53.78	77.95	73.08	77.75	79.99	0.66
Guatemala	78.86	49.71	69.33	65.09	75.96	0.92
India	9.26	79.21	52.63	63.46	67.06	0.92
Jordan	52.21	25.97	77.51	48.27	79.44	0.94
Kenya	78.97	77.58	76.14	20.41	78.87	0.95
Madagascar	67.03	77.51	71.90	24.91	73.66	0.96
Malaysia	79.24	70.39	69.88	22.49	79.33	0.99
Mauritius	77.54	54.07	76.52	36.53	78.07	0.35
Mexico	57.03	78.45	65.58	65.82	78.48	0.86
Morocco	79.49	53.13	18.97	2.51	61.73	0.98
Pakistan	73.64	73.79	31.88	78.95	79.78	0.93
Paraguay	70.19	77.97	42.44	64.83	79.01	0.95
Peru	76.09	52.10	67.48	45.57	78.57	0.95
Philippines	77.86	68.18	69.23	58.67	76.41	0.92
Senegal	75.21	69.08	68.62	77.37	78.88	0.95
South Africa	4.78	78.78	14.69	4.88	72.27	0.99
Thailand	74.49	79.94	79.12	19.93	60.09	0.98
Türkiye	79.31	54.40	48.94	72.74	73.70	0.98

Table 8: Country-Specific Estimates of the Capital Adjustment Cost Parameters and the Debt Elasticity of the Interest Rate corresponding to the Commodity Export-Price shocks

Country	ϕ_{m_c}	$\phi_{m_{\bar{c}}}$	ϕ_{x_c}	$\phi_{x_{\bar{c}}}$	ϕ_n	ψ
Algeria	57.76	78.24	65.62	28.96	77.39	0.95
Brazil	76.00	38.27	54.97	64.48	77.69	0.96
Burundi	6.89	74.07	49.87	49.62	74.35	0.98
Cameroon	71.90	18.28	72.45	73.90	63.57	0.77
Central Afr. Rep.	60.40	77.14	54.95	60.25	77.99	0.99
Colombia	38.61	78.34	15.04	8.53	75.54	0.99
Costa Rica	52.22	75.65	72.34	75.79	74.66	0.68
Dominican Rep.	26.65	4.34	77.09	58.29	75.41	0.86
Egypt	64.19	11.61	58.63	62.95	62.60	0.01
El Salvador	3.86	64.94	74.16	78.39	70.07	0.54
Ghana	11.92	76.92	75.72	70.79	75.61	0.39
Guatemala	77.99	79.19	27.99	69.45	79.52	0.91
India	29.30	76.53	57.86	42.70	73.44	0.96
Jordan	75.59	78.71	76.31	17.26	76.69	0.95
Kenya	70.68	69.15	74.49	55.55	73.46	0.94
Madagascar	34.81	79.66	74.67	54.39	68.39	0.99
Malaysia	68.74	36.69	56.51	65.14	72.47	0.94
Mauritius	78.50	45.45	69.18	53.60	74.53	0.27
Mexico	42.71	75.45	31.01	78.58	74.49	0.95
Morocco	79.81	51.97	58.40	0.20	67.60	0.90
Pakistan	54.13	77.68	74.72	46.42	76.15	0.69
Paraguay	43.49	3.74	78.21	63.04	76.35	0.96
Peru	41.45	75.96	79.19	34.55	77.99	0.87
Philippines	62.97	77.87	55.90	44.65	79.06	0.98
Senegal	32.06	50.76	75.47	76.67	76.91	0.12
South Africa	5.03	77.21	32.62	70.84	79.18	0.99
Thailand	60.91	77.97	77.03	71.12	69.21	0.93
Türkiye	78.98	41.00	79.98	55.50	76.92	0.95

A.3 Derivation of Equation (26)

The sequential budget constraint is:

$$c_t + \sum_{j \in S} i_t^j + \sum_{j \in S} \Phi_j(k_{t+1}^j - k_t^j) + p_t^\tau d_t = p_t^\tau \frac{d_{t+1}}{1+r_t} + \sum_{j \in S} w_t^j h_t^j + \sum_{j \in S} u_t^j k_t^j, \tag{A.1}$$

Combining (A.1) with Eq (20) and the definition of the profit of firms producing final goods, we have

$$p_t^\tau a_t^\tau + p_t^n a_t^n + p_t^\tau d_t = p_t^\tau \frac{d_{t+1}}{1+r_t} + \sum_{j \in S} w_t^j h_t^j + \sum_{j \in S} u_t^j k_t^j, \tag{A.2}$$

Combining (A.2) with Eq (14) and the definition of the profit of firms producing the tradable composite goods, with have

$$p_t^m a_t^m + p_t^x a_t^x + p_t^n a_t^n + p_t^\tau d_t = p_t^\tau \frac{d_{t+1}}{1+r_t} + \sum_{j \in S} w_t^j h_t^j + \sum_{j \in S} u_t^j k_t^j, \tag{A.3}$$

Combining (A.3) with Eq (16) and the definitions of the profits of firms producing the importable and exportable composite goods, we have

$$p_t^{m_c} a_t^{m_c} + p_t^{m_{\bar{c}}} a_t^{m_{\bar{c}}} + p_t^{x_c} a_t^{x_c} + p_t^{x_{\bar{c}}} a_t^{x_{\bar{c}}} + p_t^n a_t^n + p_t^\tau d_t = p_t^\tau \frac{d_{t+1}}{1+r_t} + \sum_{j \in S} w_t^j h_t^j + \sum_{j \in S} u_t^j k_t^j, \tag{A.4}$$

or, in simplified form:

$$\sum_{j \in S} p_t^j a_t^j + p_t^\tau d_t = p_t^\tau \frac{d_{t+1}}{1+r_t} + \sum_{j \in S} w_t^j h_t^j + \sum_{j \in S} u_t^j k_t^j, \tag{A.5}$$

Combining (A.5) with Eq (18) and the definitions of the profits of firms producing intermediate goods, we have

$$\sum_{j \in S} p_t^j a_t^j + p_t^\tau d_t = p_t^\tau \frac{d_{t+1}}{1+r_t} + \sum_{j \in S} p_t^j y_t^j \tag{A.6}$$

$$\implies \sum_{j \in S} p_t^j (a_t^j - y_t^j) + p_t^\tau d_t = p_t^\tau \frac{d_{t+1}}{1+r_t} \tag{A.7}$$

Combining (A.7) with Eq (21), (22), and (24) yields:

$$\implies m_t^c + m_t^{\bar{c}} - x_t^c - x_t^{\bar{c}} + 0 + p_t^\tau d_t = p_t^\tau \frac{d_{t+1}}{1+r_t} \tag{A.8}$$

$$\implies p_t^\tau \frac{d_{t+1}}{1+r_t} = p_t^\tau d_t + m_t - x_t \tag{A.9}$$

A.4 Summary of Equilibrium Conditions

$$\lambda_t = \left(c_t - \sum_{j \in S} \frac{(h_t^j)^{\omega_j}}{\omega_j} \right)^{-\sigma} \tag{A.10}$$

$$w_t^j = (h_t^j)^{\omega^j - 1} \tag{A.11}$$

$$\lambda_t p_t^\tau = \beta(1 + r_t) E_t \lambda_{t+1} p_{t+1}^\tau \tag{A.12}$$

$$\lambda_t \left[1 + \phi_j (k_{t+1}^j - k_t^j) \right] = \beta E_t \lambda_{t+1} \left[u_{t+1}^j + 1 - \delta + \phi_j (k_{t+2}^j - k_{t+1}^j) \right] \tag{A.13}$$

$$k_{t+1}^j = (1 - \delta) k_t^j + i_t^j \tag{A.14}$$

$$p_t^{m_c} = \chi_{m,c} p_t^m (a_t^{m_c})^{-\frac{1}{\mu_m}} \left[\chi_{m,c} (a_t^{m_c})^{1 - \frac{1}{\mu_m}} + (1 - \chi_{m,c})(a_t^{m_{\bar{c}}})^{1 - \frac{1}{\mu_m}} \right]^{\frac{1}{1 - \frac{1}{\mu_m}} - 1} \tag{A.15}$$

$$p_t^{m_{\bar{c}}} = (1 - \chi_{m,c}) p_t^m (a_t^{m_{\bar{c}}})^{-\frac{1}{\mu_m}} \left[\chi_{m,c} (a_t^{m_c})^{1 - \frac{1}{\mu_m}} + (1 - \chi_{m,c})(a_t^{m_{\bar{c}}})^{1 - \frac{1}{\mu_m}} \right]^{\frac{1}{1 - \frac{1}{\mu_m}} - 1} \tag{A.16}$$

$$p_t^{x_c} = \chi_{x,c} p_t^x (a_t^{x_c})^{-\frac{1}{\mu_x}} \left[\chi_{x,c} (a_t^{x_c})^{1 - \frac{1}{\mu_x}} + (1 - \chi_{x,c})(a_t^{x_{\bar{c}}})^{1 - \frac{1}{\mu_x}} \right]^{\frac{1}{1 - \frac{1}{\mu_x}} - 1} \tag{A.17}$$

$$p_t^{x_{\bar{c}}} = (1 - \chi_{x,c}) p_t^x (a_t^{x_{\bar{c}}})^{-\frac{1}{\mu_x}} \left[\chi_{x,c} (a_t^{x_c})^{1 - \frac{1}{\mu_x}} + (1 - \chi_{x,c})(a_t^{x_{\bar{c}}})^{1 - \frac{1}{\mu_x}} \right]^{\frac{1}{1 - \frac{1}{\mu_x}} - 1} \tag{A.18}$$

$$p_t^m = \chi_m p_t^\tau (a_t^m)^{-\frac{1}{\mu_{mx}}} \left[\chi_m (a_t^m)^{1 - \frac{1}{\mu_{mx}}} + (1 - \chi_m)(a_t^x)^{1 - \frac{1}{\mu_{mx}}} \right]^{\frac{1}{1 - \frac{1}{\mu_{mx}}} - 1} \tag{A.19}$$

$$p_t^x = (1 - \chi_m) p_t^\tau (a_t^x)^{-\frac{1}{\mu_{mx}}} \left[\chi_m (a_t^m)^{1 - \frac{1}{\mu_{mx}}} + (1 - \chi_m)(a_t^x)^{1 - \frac{1}{\mu_{mx}}} \right]^{\frac{1}{1 - \frac{1}{\mu_{mx}}} - 1} \tag{A.20}$$

$$p_t^\tau = \chi_\tau (a_t^\tau)^{-\frac{1}{\mu_{\tau n}}} \left[\chi_\tau (a_t^\tau)^{1 - \frac{1}{\mu_{\tau n}}} + (1 - \chi_\tau)(a_t^n)^{1 - \frac{1}{\mu_{\tau n}}} \right]^{\frac{1}{1 - \frac{1}{\mu_{\tau n}}} - 1} \tag{A.21}$$

$$p_t^n = (1 - \chi_\tau)(a_t^n)^{-\frac{1}{\mu_{\tau n}}} \left[\chi_\tau (a_t^\tau)^{1 - \frac{1}{\mu_{\tau n}}} + (1 - \chi_\tau)(a_t^n)^{1 + \frac{1}{\mu_{\tau n}}} \right]^{\frac{1}{1 - \frac{1}{\mu_{\tau n}}} - 1} \tag{A.22}$$

$$y_t^j = Z_t^j (k_t^j)^{\alpha^j} (h_t^j)^{1 - \alpha^j} \tag{A.23}$$

$$\alpha^j p_t^j Z_t^j (k_t^j)^{\alpha^j - 1} (h_t^j)^{1 - \alpha^j} = u_t^j \tag{A.24}$$

$$w_t^j = (1 - \alpha^j) p_t^j Z_t^j (k_t^j)^{\alpha^j} (h_t^j)^{-\alpha^j} \tag{A.25}$$

$$c_t + \sum_{j \in S} i_t^j + \sum_{j \in S} \frac{\phi_j}{2} \left(k_{t+1}^j - k_t^j \right)^2 = \left[\chi_\tau (a_t^\tau)^{1 - \frac{1}{\mu_{\tau n}}} + (1 - \chi_\tau)(a_t^n)^{1 - \frac{1}{\mu_{\tau n}}} \right]^{\frac{1}{1 - \frac{1}{\mu_{\tau n}}}} \tag{A.26}$$

$$a_t^\tau = \left[\chi_m (a_t^m)^{1-\frac{1}{\mu_{mx}}} + (1-\chi_m)(a_t^x)^{1-\frac{1}{\mu_{mx}}}\right]^{\frac{1}{1-\frac{1}{\mu_{mx}}}} \tag{A.27}$$

$$a_t^m = \left[\chi_{m,c} (a_t^{m_c})^{1-\frac{1}{\mu_m}} + (1-\chi_{m,c})(a_t^{m_{\bar{c}}})^{1-\frac{1}{\mu_m}}\right]^{\frac{1}{1-\frac{1}{\mu_m}}} \tag{A.28}$$

$$a_t^x = \left[\chi_{x,c} (a_t^{x_c})^{1-\frac{1}{\mu_x}} + (1-\chi_{x,c})(a_t^{x_{\bar{c}}})^{1-\frac{1}{\mu_x}}\right]^{\frac{1}{1-\frac{1}{\mu_x}}} \tag{A.29}$$

$$m_t^c = p_t^{m_c}\left(a_t^{m_c} - y_t^{m_c}\right) \tag{A.30}$$

$$m_t^{\bar{c}} = p_t^{m_{\bar{c}}}\left(a_t^{m_{\bar{c}}} - y_t^{m_{\bar{c}}}\right) \tag{A.31}$$

$$x_t^c = p_t^{x_c}\left(y_t^{x_c} - a_t^{x_c}\right) \tag{A.32}$$

$$x_t^{x_{\bar{c}}} = p_t^{x_{\bar{c}}}\left(y_t^{x_{\bar{c}}} - a_t^{x_{\bar{c}}}\right) \tag{A.33}$$

$$m_t = m_t^c + m_t^{\bar{c}} \tag{A.34}$$

$$x_t = x_t^c + x_t^{\bar{c}} \tag{A.35}$$

$$a_t^n = y_t^n \tag{A.36}$$

$$p_t^\tau \frac{d_{t+1}}{1+r_t} = p_t^\tau d_t + m_t - x_t \tag{A.37}$$

$$r_t = r^* + s_t + \psi(e^{d_{t+1}-\bar{d}} - 1) \tag{A.38}$$

$$tot_t = \frac{p_t^x}{p_t^m} \tag{A.39}$$

$$tot_t^c = \frac{p_t^{x_c}}{p_t^{m_c}} \tag{A.40}$$

$$\begin{cases} \log\left(\frac{p_t^k}{p^k}\right) = \rho_{k,k}\log\left(\frac{p_{t-1}^k}{p^k}\right) + \rho_{k,s}s_t + \pi_k \epsilon_t^k, \\ s_t = \rho_{s,s}s_{t-1} + \rho_{s,k}\log\left(\frac{p_{t-1}^k}{p^k}\right) + \pi_{s,k}\epsilon_t^k + \pi_s \epsilon_t^s, \\ \log\left(\frac{p_t^{k'}}{p^{k'}}\right) = \rho_{k'}\log\left(\frac{p_{t-1}^{k'}}{p^{k'}}\right) + \pi_{k'}\epsilon_t^{k'}, \end{cases} \tag{A.41}$$

with $k' = m^c$ if $k = x^c$, and $k' = x^c$ if $k = m^c$. [4]

$$\log\left(\frac{Z_t^j}{Z^j}\right) = \rho_{z,j}\log\left(\frac{Z_t^j}{Z^j}\right) + \pi_{z,j}\epsilon_t^{z,j} \tag{A.42}$$

[4]Equation (A.41) means that the commodity import and export price shocks are analyzed separately. When the shock under consideration is the commodity import price shock, the commodity import price index follows a joint law of motion with the U.S. interest rate spread, and the commodity export price shock is not explored. And vice versa.

$$y_t = \sum_{j \in S} p_t^j y_t^j \tag{A.43}$$

$$i_t = \sum_{j \in S} i_t^j \tag{A.44}$$

$$y_t^0 = \sum_{j \in S} p^j y_t^j \tag{A.45}$$

$$c_t^0 = c_t \frac{y_t^0}{y_t} \tag{A.46}$$

$$i_t^0 = i_t \frac{y_t^0}{y_t} \tag{A.47}$$

$$tb_t^0 = (x_t - m_t) \frac{y_t^0}{y_t} \tag{A.48}$$

A.5 Steady State

From (A.12):

$$\beta = \frac{1}{1+r} \tag{A.49}$$

From (A.39):

$$r = r^* + s \tag{A.50}$$

From (A.13):

$$u^{m_c} = u^{m_{\bar{c}}} = u^{x_c} = u^{x_{\bar{c}}} = u^n = \frac{1}{\beta} - 1 + \delta \tag{A.51}$$

Using (A.15) and (A.16), we get:

$$\frac{p^{m_c}}{p^{m_{\bar{c}}}} = \frac{\chi_{m,c}}{1-\chi_{m,c}} \left(\frac{a^{m_c}}{a^{m_{\bar{c}}}} \right)^{-\frac{1}{\mu_m}} \tag{A.52}$$

This implies:

$$\chi_{m,c} = \frac{1}{1 + \frac{p^{m_{\bar{c}}}}{p^{m_c}} \left(\frac{a^{m_c}}{a^{m_{\bar{c}}}} \right)^{-\frac{1}{\mu_m}}} \tag{A.53}$$

Notice that $\frac{a^{m_c}}{a^{m_{\bar{c}}}} = \frac{p^{m_{\bar{c}}}}{p^{m_c}} \cdot \frac{s_{a_{m,c}}}{s_{a_{m,\bar{c}}}}$, where $s_{a_{m,c}} = \frac{p^{m_c} a^{m_c}}{y}$ and $s_{a_{m,\bar{c}}} = \frac{p^{m_{\bar{c}}} a^{m_{\bar{c}}}}{y}$ are, respectively, the shares of the domestic absorption of commodity importables and non-commodity importables in total output. In addition, $s_{a_{m,c}} = s_{y_{m,c}} + s_{m_c}$, where $s_{y_{m,c}}$ and s_{m_c} are, respectively, the shares of commodity importable output and commodity imports in total output; and $s_{a_{m,\bar{c}}} = s_{y_{m,\bar{c}}} + s_{m_{\bar{c}}}$, where $s_{y_{m,\bar{c}}}$ and $s_{m_{\bar{c}}}$ are, respectively, the shares of non-commodity importable output and non-commodity imports in total output. It follows that:

$$\chi_{m,c} = \frac{1}{1 + \frac{p^{m_{\bar{c}}}}{p^{m_c}} \left(\frac{p^{m_{\bar{c}}}}{p^{m_c}} \cdot \frac{s_{y_{m,c}} + s_{m_c}}{s_{y_{m,\bar{c}}} + s_{m_{\bar{c}}}} \right)^{-\frac{1}{\mu_m}}} \tag{A.54}$$

We obtain $\chi_{x,c}$ in a similar way using (A.17) and (A.18):

$$\chi_{x,c} = \frac{1}{1 + \frac{p^{x_{\bar{c}}}}{p^{x_c}} \left(\frac{p^{x_{\bar{c}}}}{p^{x_c}} \cdot \frac{s_{y_{x,c}} - s_{x_c}}{s_{y_{x,\bar{c}}} - s_{x_{\bar{c}}}} \right)^{-\frac{1}{\mu_x}}} \tag{A.55}$$

where $s_{y_{x,c}}$ and s_{x_c} are, respectively, the shares of commodity exportable output and commodity exports in total output; and $s_{y_{x,\bar{c}}}$ and $s_{x_{\bar{c}}}$ are, respectively, the shares of non-commodity exportable output and non-commodity exports in total output.

Using (A.15) we have:

$$\frac{p^{m_c}}{p^m} = \chi_{m,c} \left(\frac{a^{m_c}}{a^m} \right)^{-\frac{1}{\mu_m}} \tag{A.56}$$

Then, from (A.29), we get:

$$\frac{a^{m_c}}{a^m} = \left(\chi_{m,c} + (1 - \chi_{m,c}) \left(\frac{a^{m_c}}{a^{m_{\bar{c}}}} \right)^{\frac{1}{\mu_m} - 1} \right)^{-\frac{1}{1 - \frac{1}{\mu_m}}} = \left(\chi_{m,c} + (1 - \chi_{m,c}) \left(\frac{p^{m_{\bar{c}}}}{p^{m_c}} \cdot \frac{s_{y_{m,c}} + s_{m_c}}{s_{y_{m,\bar{c}}} + s_{m_{\bar{c}}}} \right)^{\frac{1}{\mu_m} - 1} \right)^{-\frac{1}{1 - \frac{1}{\mu_m}}} \tag{A.57}$$

This implies that the price of the composite importable good is:

$$p^m = \frac{p^{m_c}}{\chi_{m,c} \left(\frac{a^{m_c}}{a^m} \right)^{-\frac{1}{\mu_m}}} = \frac{p^{m_c}}{\chi_{m,c} \left[\left(\chi_{m,c} + (1 - \chi_{m,c}) \left(\frac{p^{m_{\bar{c}}}}{p^{m_c}} \cdot \frac{s_{y_{m,c}} + s_{m_c}}{s_{y_{m,\bar{c}}} + s_{m_{\bar{c}}}} \right)^{\frac{1}{\mu_m} - 1} \right)^{-\frac{1}{1 - \frac{1}{\mu_m}}} \right]^{-\frac{1}{\mu_m}}} \tag{A.58}$$

Similarly, using (A.17) and (A.30), we obtain the price of the composite exportable good as:

$$p^x = \frac{p^{x_c}}{\chi_{x,c} \left(\frac{a^{x_c}}{a^x} \right)^{-\frac{1}{\mu_x}}} = \frac{p^{x_c}}{\chi_{x,c} \left[\left(\chi_{x,c} + (1 - \chi_{x,c}) \left(\frac{p^{x_{\bar{c}}}}{p^{x_c}} \cdot \frac{s_{y_{x,c}} - s_{x_c}}{s_{y_{x,\bar{c}}} + s_{x_{\bar{c}}}} \right)^{\frac{1}{\mu_x} - 1} \right)^{-\frac{1}{1 - \frac{1}{\mu_x}}} \right]^{-\frac{1}{\mu_x}}} \tag{A.59}$$

From (A.24), the capital-to-labor ratio in sector j is:

$$\frac{k^j}{h^j} = \left(\frac{u^j}{\alpha^j p^j Z^j} \right)^{\frac{1}{\alpha^j - 1}} \tag{A.60}$$

Then, using (A.25), we obtain the wage rate in sector j as:

$$w^j = (1-\alpha^j)p^j Z^j \left(\frac{k^j}{h^j}\right)^{\alpha^j} = (1-\alpha^j)p^j Z^j \left(\frac{u^j}{\alpha^j p^j Z^j}\right)^{\frac{\alpha^j}{\alpha^j-1}} \tag{A.61}$$

From (A.11), labor in sector j is then given by:

$$h^j = \left(w^j\right)^{\frac{1}{\omega^j-1}} = \left[(1-\alpha^j)p^j Z^j \left(\frac{u^j}{\alpha^j p^j Z^j}\right)^{\frac{\alpha^j}{\alpha^j-1}}\right]^{\frac{1}{\omega^j-1}} \tag{A.62}$$

Now, using (A.61), we have:

$$k^j = \left(\frac{u^j}{\alpha^j p^j Z^j}\right)^{\frac{1}{\alpha^j-1}} h^j = \left(\frac{u^j}{\alpha^j p^j Z^j}\right)^{\frac{1}{\alpha^j-1}} \left[(1-\alpha^j)p^j Z^j \left(\frac{u^j}{\alpha^j p^j Z^j}\right)^{\frac{\alpha^j}{\alpha^j-1}}\right]^{\frac{1}{\omega^j-1}} \tag{A.63}$$

Using (A.23), output in sector j is given by:

$$y^j = Z^j (k^j)^{\alpha^j} (h^j)^{1-\alpha^j} \tag{A.64}$$

From (A.14), investment in sector j is:

$$i^j = \delta k^j \tag{A.65}$$

From (A.37):

$$a^n = y^n \tag{A.66}$$

From (A.44)

$$y = \sum_{j \in S} p^j y^j \tag{A.67}$$

Imports and exports of commodity and non-commodity goods are given by:

$$m^c = s_{m_c} y \; ; \qquad m^{\bar{c}} = s_{m_{\bar{c}}} y \tag{A.68}$$

$$x^c = s_{x_c} y \; ; \qquad x^{\bar{c}} = x_{m_{\bar{c}}} y \tag{A.69}$$

Total imports and exports are given by:

$$m = m^c + m^{\bar{c}} \; ; \qquad x = x^c + x^{\bar{c}} \tag{A.70}$$

Using (A.31) to (A.34), we have:

$$a^{m_c} = y^{m_c} + \frac{m_c}{p^{m_c}}\,; \qquad a^{m_{\bar{c}}} = y^{m_{\bar{c}}} + \frac{m_{\bar{c}}}{p^{m_{\bar{c}}}} \tag{A.71}$$

$$a^{x_c} = y^{x_c} - \frac{x_c}{p^{x_c}}\,; \qquad a^{x_{\bar{c}}} = y^{x_{\bar{c}}} + \frac{x_{\bar{c}}}{p^{x_{\bar{c}}}} \tag{A.72}$$

Using (A.19) and (A.20), we have:

$$\frac{p^x}{p^m} = \frac{1-\chi_m}{\chi_m}\left(\frac{a^x}{a^m}\right)^{-\frac{1}{\mu_{mx}}} \implies \chi_m = \frac{1}{1+\frac{p^x}{p^m}\left(\frac{a^x}{a^m}\right)^{\frac{1}{\mu_{mx}}}} \tag{A.73}$$

where

$$a^m = \left[\chi_{m,c}(a^{m_c})^{1-\frac{1}{\mu_m}} + (1-\chi_{m,c})(a^{m_{\bar{c}}})^{1-\frac{1}{\mu_m}}\right]^{\frac{1}{1-\frac{1}{\mu_m}}} \tag{A.74}$$

and

$$a^x = \left[\chi_{x,c}(a^{x_c})^{1-\frac{1}{\mu_x}} + (1-\chi_{x,c})(a^{x_{\bar{c}}})^{1-\frac{1}{\mu_x}}\right]^{\frac{1}{1-\frac{1}{\mu_x}}} \tag{A.75}$$

Using (A.19) we have:

$$\frac{p^m}{p^\tau} = \chi_m\left(\frac{a^m}{a^\tau}\right)^{-\frac{1}{\mu_{mx}}} \tag{A.76}$$

Then, from (A.28), we get:

$$\frac{a^m}{a^\tau} = \left(\chi_m + (1-\chi_m)\left(\frac{a^m}{a^x}\right)^{\frac{1}{\mu_{mx}}-1}\right)^{-\frac{1}{1-\frac{1}{\mu_{mx}}}} \tag{A.77}$$

This implies that:

$$p^\tau = \frac{p^m}{\chi_m\left(\frac{a^m}{a^\tau}\right)^{-\frac{1}{\mu_{mx}}}} = \frac{p^m}{\chi_m\left[\left(\chi_m + (1-\chi_m)\left(\frac{a^m}{a^x}\right)^{\frac{1}{\mu_{mx}}-1}\right)^{-\frac{1}{1-\frac{1}{\mu_{mx}}}}\right]^{-\frac{1}{\mu_{mx}}}} \tag{A.78}$$

Using (A.38), we have:

$$d = \frac{(1+r)(x-m)}{rp^\tau} \tag{A.79}$$

From (A.21) and (A.22), we have

$$\frac{p^n}{p^\tau} = \frac{1-\chi_\tau}{\chi_\tau}\left(\frac{a^n}{a^\tau}\right)^{-\frac{1}{\mu_{\tau n}}} \implies \chi_\tau = \frac{1}{1+\frac{p^n}{p^\tau}\left(\frac{a^n}{a^\tau}\right)^{\frac{1}{\mu_{\tau n}}}} \tag{A.80}$$

where

$$a^{\tau} = \left[\chi_m (a^m)^{1-\frac{1}{\mu_{mx}}} + (1-\chi_m)(a^x)^{1-\frac{1}{\mu_{mx}}} \right]^{\frac{1}{1-\frac{1}{\mu_{mx}}}} \tag{A.81}$$

From (A.26), we have:

$$c = \left[\chi_\tau (a^{\tau})^{1-\frac{1}{\mu_{\tau n}}} + (1-\chi_\tau)(a^n)^{1-\frac{1}{\mu_{\tau n}}} \right]^{\frac{1}{1-\frac{1}{\mu_{\tau n}}}} - \sum_{j \in S} i^j \tag{A.82}$$

The value of λ is given by (A.10):

$$\lambda = \left(c - \sum_{j \in S} \frac{(h^j)^{\omega_j}}{\omega_j} \right)^{-\sigma} \tag{A.83}$$

Finally, equations (A.45) to (A.48) give the theoretical counterparts of the observed measures of real GDP, real consumption, real investment, and the trade balance-to-GDP ratio:

$$y^0 = \sum_{j \in S} p^j y^j = y \tag{A.84}$$

$$c^0 = c \frac{y^0}{y} = c \tag{A.85}$$

$$i^0 = i \frac{y^0}{y} = i \tag{A.86}$$

$$tb^y = \frac{tb^0}{y} = \frac{x-m}{y} \cdot \frac{y^0}{y} = \frac{x-m}{y} \tag{A.87}$$